Graph algorithms recap through the complexity lens

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Practice problems

1. A graph has $V = 5000$ vertices and $E = 20000$ edges. Calculate the approximate number of operations for B-F-S using $O(V + E)$ and for Dijkstra's using $O(E \log_2 V)$.

2. In a complete graph with V vertices, the number of edges is $E = \frac{V(V-1)}{2}$. What is the Big-O complexity of B-F-S in terms of V only for a complete graph?

3. You have a choice between an algorithm that runs in $O(V^2)$ and one that runs in $O(E \log V)$. If the graph is very sparse ($E \approx V$), which one is better as V becomes very large?

Solutions

1. A graph has $V = 5000$ vertices and $E = 20000$ edges. Calculate the approximate number of operations for B-F-S using $O(V + E)$ and for Dijkstra's using $O(E \log_2 V)$.

   - For B-F-S, the cost is $5000 + 20000 = 25000$ operations.
   - For Dijkstra's, $\log_2(5000)$ is approximately 12.28.
   - The cost is $20000 \times 12.28 \approx 245600$ operations.

   Answer: B-F-S: 25,000; Dijkstra: $\approx 245,600$

2. In a complete graph with V vertices, the number of edges is $E = \frac{V(V-1)}{2}$. What is the Big-O complexity of B-F-S in terms of V only for a complete graph?

   - The general cost is $O(V + E)$.
   - Substitute $E = \frac{V^2-V}{2}$.
   - The expression becomes $O(V + \frac{V^2-V}{2})$, which is dominated by the V^2 term.

   Answer: O(V^2)

3. You have a choice between an algorithm that runs in $O(V^2)$ and one that runs in $O(E \log V)$. If the graph is very sparse ($E \approx V$), which one is better as V becomes very large?

   - If $E \approx V$, the second algorithm runs in $O(V \log V)$.
   - We compare V^2 with $V \log V$.
   - Since $\log V$ grows much slower than V, $V \log V$ is asymptotically smaller than V^2.

   Answer: O(E \log V)