Counting steps: Big-O honestly defined

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Practice problems

1. Prove that $f(n) = 7n^3 + 12n^2 - 5$ is $O(n^3)$ by finding specific values for C and n_0.
2. Use the limit test to determine if $f(n) = n^2 \log n$ is $O(n^3)$.
3. Determine if $f(n) = 3^n$ is $O(n^{100})$. Justify your answer.

Solutions

1. Prove that $f(n) = 7n^3 + 12n^2 - 5$ is $O(n^3)$ by finding specific values for C and n_0.

   - For $n \geq 1$, we know $n^2 \leq n^3$ and $1 \leq n^3$.
   - Thus, $7n^3 + 12n^2 - 5 \leq 7n^3 + 12n^3 + 0$.
   - This simplifies to $19n^3$.
   - We can choose $C = 19$ and $n_0 = 1$.

   Answer: C = 19, n_0 = 1 (or any valid pair)

2. Use the limit test to determine if $f(n) = n^2 \log n$ is $O(n^3)$.

   - Compute the limit $\lim_{n\to\infty} \frac{n^2 \log n}{n^3}$.
   - Simplify the expression to $\lim_{n\to\infty} \frac{\log n}{n}$.
   - Apply L'Hopital's rule: the derivative of $\log n$ is $1/n$ and the derivative of n is 1.
   - The limit is $\lim_{n\to\infty} \frac{1}{n} = 0$.
   - Since $0 < \infty$, the relation holds.

   Answer: Yes, f(n) = O(n^3)

3. Determine if $f(n) = 3^n$ is $O(n^{100})$. Justify your answer.

   - Consider the limit $\lim_{n\to\infty} \frac{3^n}{n^{100}}$.
   - Applying L'Hopital's rule 100 times, the denominator becomes a constant (100!).
   - The numerator remains a multiple of 3^n (specifically $(\ln 3)^{100} 3^n$).
   - The limit is ∞.
   - Since the limit is not finite, 3^n grows strictly faster than n^{100}.

   Answer: No, f(n) is not O(n^100)