Module 6 Review and Self-Test

Lesson 84 · Module 6 — Graph Theory: Structure & Colour · Discrete Mathematics · daily-maths.uk/discrete

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Practice problems

1. A graph has 6 vertices with degrees $3, 3, 4, 4, 2, 2$. How many edges does it have?
2. A connected planar graph has 10 vertices and 15 edges. How many faces does it have?
3. What is the chromatic number of the complete graph K_4?

Solutions

1. A graph has 6 vertices with degrees $3, 3, 4, 4, 2, 2$. How many edges does it have?

   - Sum the degrees: $3 + 3 + 4 + 4 + 2 + 2 = 18$.
   - Apply the handshake lemma: $2|E| = 18$.
   - Divide by 2 to find $|E| = 9$.

   Answer: 9

2. A connected planar graph has 10 vertices and 15 edges. How many faces does it have?

   - Use Euler's formula: $V - E + F = 2$.
   - Substitute values: $10 - 15 + F = 2$.
   - Simplify: $-5 + F = 2$.
   - Solve for F: $F = 7$.

   Answer: 7

3. What is the chromatic number of the complete graph K_4?

   - In K_4, every vertex is connected to every other vertex.
   - Vertex 1 gets colour 1.
   - Vertex 2 is adjacent to 1, so it gets colour 2.
   - Vertex 3 is adjacent to 1 and 2, so it gets colour 3.
   - Vertex 4 is adjacent to 1, 2, and 3, so it must get colour 4.

   Answer: 4