

Ramsey numbers on graphs; $R(3,3) = 6$ revisited graphically

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Practice problems

1. Using the recursive bound $R(r, s) \leq R(r-1, s) + R(r, s-1)$, and given that $R(2, 5) = 5$ and $R(3, 4) = 9$, find an upper bound for $R(3, 5)$.
2. Explain why $R(2, s) = s$ for any integer $s \geq 2$.
3. A group of 6 people are at a party. Prove that there must be either 3 people who all know each other or 3 people who are all strangers to each other.

Solutions

1. Using the recursive bound $R(r, s) \leq R(r - 1, s) + R(r, s - 1)$, and given that $R(2, 5) = 5$ and $R(3, 4) = 9$, find an upper bound for $R(3, 5)$.

- Apply the recursive formula: $R(3, 5) \leq R(2, 5) + R(3, 4)$.
- Substitute the given values: $R(3, 5) \leq 5 + 9$.
- Calculate the sum: $5 + 9 = 14$.

Answer: 14

2. Explain why $R(2, s) = s$ for any integer $s \geq 2$.

- Consider a complete graph K_s .
- If any edge is coloured red, we have a monochromatic K_2 in red, satisfying the condition.
- If no edges are coloured red, then all edges must be blue.
- Since the graph is K_s , all edges being blue means we have a monochromatic K_s in blue.
- Thus, s vertices are always sufficient. For $s - 1$ vertices, we can colour all edges blue, which contains no red K_2 and no blue K_s .

Answer: $R(2, s) = s$

3. A group of 6 people are at a party. Prove that there must be either 3 people who all know each other or 3 people who are all strangers to each other.

- Model the people as vertices of a complete graph K_6 .
- Colour an edge red if two people know each other, and blue if they are strangers.
- The problem is equivalent to finding a monochromatic K_3 in a 2-coloured K_6 .
- Since $R(3, 3) = 6$, any 2-colouring of K_6 must contain a monochromatic K_3 .
- Therefore, there must be a red triangle (3 mutual acquaintances) or a blue triangle (3 mutual strangers).

Answer: Proven by $R(3, 3) = 6$