

Network flows and max-flow/min-cut — a first look

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Practice problems

1. Given a network with capacities $c(s, A) = 10, c(s, B) = 5, c(A, B) = 2, c(A, t) = 8, c(B, t) = 7$, find the maximum flow from s to t .
2. In a network, if the minimum cut capacity is 12, what is the maximum possible flow that can be sent from the source to the sink?
3. Consider a network where the source s connects to A and B with capacity 10 each. A connects to t with capacity 5, and B connects to t with capacity 5. There is an edge from A to B with capacity 10. Find the max flow.

Solutions

1. Given a network with capacities $c(s, A) = 10, c(s, B) = 5, c(A, B) = 2, c(A, t) = 8, c(B, t) = 7$, find the maximum flow from s to t .
 - Path $s \rightarrow A \rightarrow t$ can take $\min(10, 8) = 8$ units.
 - Path $s \rightarrow B \rightarrow t$ can take $\min(5, 7) = 5$ units.
 - Remaining capacity $c(s, A) = 2, c(s, B) = 0, c(A, B) = 2, c(A, t) = 0, c(B, t) = 2$.
 - Path $s \rightarrow A \rightarrow B \rightarrow t$ can take $\min(2, 2, 2) = 2$ units.
 - Total flow is $8 + 5 + 2 = 15$.
 - Check cut $S = \{s, A, B\}, T = \{t\}$: capacity is $c(A, t) + c(B, t) = 8 + 7 = 15$.

Answer: 15

2. In a network, if the minimum cut capacity is 12, what is the maximum possible flow that can be sent from the source to the sink?
 - According to the Max-Flow Min-Cut Theorem, the maximum flow value is exactly equal to the capacity of the minimum cut.
 - Since the minimum cut capacity is given as 12, the max flow must also be 12.

Answer: 12

3. Consider a network where the source s connects to A and B with capacity 10 each. A connects to t with capacity 5, and B connects to t with capacity 5. There is an edge from A to B with capacity 10. Find the max flow.
 - Path $s \rightarrow A \rightarrow t$ takes 5 units.
 - Path $s \rightarrow B \rightarrow t$ takes 5 units.
 - Total flow is 10.
 - Check cut $S = \{s, A, B\}, T = \{t\}$: capacity is $c(A, t) + c(B, t) = 5 + 5 = 10$.
 - Since flow equals cut capacity, the max flow is 10.

Answer: 10