

K5 and K33 cannot be drawn flat

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Practice problems

1. A bipartite graph has $V = 8$ vertices and $E = 13$ edges. Is it possible for this graph to be planar?
2. A simple graph has $V = 6$ vertices and $E = 13$ edges. Prove that it is non-planar.
3. Consider a bipartite graph with $V = 10$ vertices. What is the maximum number of edges it can have while still potentially being planar?

Solutions

1. A bipartite graph has $V = 8$ vertices and $E = 13$ edges. Is it possible for this graph to be planar?

- Identify the graph as bipartite, so use the inequality $E \leq 2V - 4$.
- Substitute the values: $13 \leq 2(8) - 4$.
- Calculate the right side: $2(8) - 4 = 16 - 4 = 12$.
- Check the inequality: $13 \leq 12$ is false.

Answer: No, it cannot be planar.

2. A simple graph has $V = 6$ vertices and $E = 13$ edges. Prove that it is non-planar.

- Use the general planar inequality $E \leq 3V - 6$.
- Substitute the values: $13 \leq 3(6) - 6$.
- Calculate the right side: $18 - 6 = 12$.
- Check the inequality: $13 \leq 12$ is false.

Answer: The graph is non-planar because it exceeds the maximum number of edges allowed for a planar graph with 6 vertices.

3. Consider a bipartite graph with $V = 10$ vertices. What is the maximum number of edges it can have while still potentially being planar?

- Use the bipartite planar inequality $E \leq 2V - 4$.
- Substitute $V = 10$: $E \leq 2(10) - 4$.
- Calculate: $E \leq 20 - 4 = 16$.

Answer: 16 edges