Planar Graphs and Euler's Formula

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Practice problems

1. A connected planar graph has $V = 12$ vertices and $E = 20$ edges. How many faces F does it have?

2. A planar graph consists of 3 separate connected components. The total number of vertices is $V = 15$ and the total number of edges is $E = 12$. How many faces F does this graph have?

3. A connected planar graph has $V = 8$ vertices and $F = 6$ faces. How many edges E does it have?

Solutions

1. A connected planar graph has $V = 12$ vertices and $E = 20$ edges. How many faces F does it have?

   - Use Euler's formula for connected planar graphs: $V - E + F = 2$.
   - Substitute the given values: $12 - 20 + F = 2$.
   - Simplify the left side: $-8 + F = 2$.
   - Solve for F: $F = 2 + 8 = 10$.

   Answer: 10

2. A planar graph consists of 3 separate connected components. The total number of vertices is $V = 15$ and the total number of edges is $E = 12$. How many faces F does this graph have?

   - Use the generalized Euler's formula for C components: $V - E + F = 1 + C$.
   - Substitute the values: $15 - 12 + F = 1 + 3$.
   - Simplify: $3 + F = 4$.
   - Solve for F: $F = 4 - 3 = 1$.

   Answer: 1

3. A connected planar graph has $V = 8$ vertices and $F = 6$ faces. How many edges E does it have?

   - Use Euler's formula for connected planar graphs: $V - E + F = 2$.
   - Substitute the given values: $8 - E + 6 = 2$.
   - Simplify the left side: $14 - E = 2$.
   - Solve for E: $E = 14 - 2 = 12$.

   Answer: 12