

Module 5 Review and Self-Test

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Practice problems

1. Determine if a graph exists with the degree sequence $3, 3, 3, 2, 2, 1$. If so, how many edges does it have?
2. A weighted graph has vertices A, B, C, D and edges $(A, B, 2), (B, C, 3), (C, D, 1), (D, A, 4), (A, C, 5)$. Find the total weight of the Minimum Spanning Tree using Kruskal's algorithm.
3. In an unweighted graph, you start at node S . You want to find the shortest path to node T . Which algorithm is more efficient for this specific goal: BFS or DFS? Explain why.

Solutions

1. Determine if a graph exists with the degree sequence $3, 3, 3, 2, 2, 1$. If so, how many edges does it have?

- Sum the degrees: $3 + 3 + 3 + 2 + 2 + 1 = 14$.
- Check if the sum is even: 14 is even, so the handshake lemma is satisfied.
- Calculate edges: $|E| = 14/2 = 7$.

Answer: Yes, it exists with 7 edges.

2. A weighted graph has vertices A, B, C, D and edges $(A, B, 2), (B, C, 3), (C, D, 1), (D, A, 4), (A, C, 5)$. Find the total weight of the Minimum Spanning Tree using Kruskal's algorithm.

- Sort edges: $(C, D, 1), (A, B, 2), (B, C, 3), (D, A, 4), (A, C, 5)$.
- Pick $(C, D, 1)$.
- Pick $(A, B, 2)$.
- Pick $(B, C, 3)$. Now we have 3 edges for 4 vertices, and no cycles.
- Total weight: $1 + 2 + 3 = 6$.

Answer: 6

3. In an unweighted graph, you start at node S . You want to find the shortest path to node T . Which algorithm is more efficient for this specific goal: BFS or DFS? Explain why.

- BFS explores the graph in layers (breadth-first).
- The first time BFS reaches node T , it is guaranteed to have taken the path with the minimum number of edges.
- DFS explores as deep as possible and may find a very long path to T before ever finding the shortest one.
- Therefore, BFS is the correct choice for shortest paths in unweighted graphs.

Answer: BFS, because it explores layer by layer and guarantees the shortest path in unweighted graphs.