

Trees II — Cayley's formula via the Prufer bijection

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Practice problems

1. Find the Prufer sequence for the path graph on 6 vertices with edges $(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)$.
2. How many distinct labelled trees can be formed using 6 vertices?
3. A labelled tree on 5 vertices has a Prufer sequence $(3, 3, 1)$. What is the degree of vertex 3 in this tree?

Solutions

1. Find the Prufer sequence for the path graph on 6 vertices with edges $(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)$.

- Smallest leaf is 1, neighbour is 2. Sequence: (2) , Tree: $2, 3, 4, 5, 6$
- Smallest leaf is 2, neighbour is 3. Sequence: $(2, 3)$, Tree: $3, 4, 5, 6$
- Smallest leaf is 3, neighbour is 4. Sequence: $(2, 3, 4)$, Tree: $4, 5, 6$
- Smallest leaf is 4, neighbour is 5. Sequence: $(2, 3, 4, 5)$, Tree: $5, 6$
- Stop when 2 vertices remain.

Answer: $(2, 3, 4, 5)$

2. How many distinct labelled trees can be formed using 6 vertices?

- Use Cayley's formula $T_n = n^{n-2}$
- Substitute $n = 6$
- Compute $6^{6-2} = 6^4$
- $6^4 = 1296$

Answer: 1296

3. A labelled tree on 5 vertices has a Prufer sequence $(3, 3, 1)$. What is the degree of vertex 3 in this tree?

- Use the property that the number of occurrences of vertex v in the sequence is $\deg(v) - 1$.
- Vertex 3 appears 2 times in the sequence $(3, 3, 1)$.
- Set $2 = \deg(3) - 1$.
- Solve for $\deg(3) = 3$.

Answer: 3