

Eulerian paths and circuits — Königsberg solved

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Practice problems

1. A connected graph has 5 vertices with degrees 2, 4, 2, 2, and 2. Does it have an Eulerian circuit? Does it have an Eulerian path?
2. A connected graph has 6 vertices. Four vertices have degree 3 and two vertices have degree 2. Does it have an Eulerian path?
3. Consider a complete graph K_5 . Does it contain an Eulerian circuit? Justify your answer.

Solutions

1. A connected graph has 5 vertices with degrees 2, 4, 2, 2, and 2. Does it have an Eulerian circuit? Does it have an Eulerian path?

- Check the degrees of all vertices: 2, 4, 2, 2, 2.
- Observe that every single degree is an even number.
- According to the theorem, if all vertices in a connected graph have even degrees, an Eulerian circuit exists.
- Since every Eulerian circuit is also an Eulerian path, both exist.

Answer: Yes to both.

2. A connected graph has 6 vertices. Four vertices have degree 3 and two vertices have degree 2. Does it have an Eulerian path?

- Count the number of odd-degree vertices.
- The vertices with degree 3 are odd. There are 4 such vertices.
- The theorem states an Eulerian path exists if and only if there are 0 or 2 odd-degree vertices.
- Since there are 4 odd-degree vertices, no Eulerian path exists.

Answer: No.

3. Consider a complete graph K_5 . Does it contain an Eulerian circuit? Justify your answer.

- In a complete graph K_n , every vertex is connected to every other vertex, so $\deg(v) = n - 1$.
- For K_5 , $n = 5$, so $\deg(v) = 5 - 1 = 4$ for all vertices.
- Since all vertices have a degree of 4, which is even, and the graph is connected, it satisfies the condition for an Eulerian circuit.

Answer: Yes, because all vertices have degree 4.