

Special graphs bestiary

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Practice problems

1. Calculate the number of edges in the complete bipartite graph $B_{5,7}$.
2. A graph G is a hypercube Q_4 . How many vertices does it have, and what is the degree of each vertex?
3. Determine the number of edges in a complete graph K_8 and verify it using the Handshake Lemma.

Solutions

1. Calculate the number of edges in the complete bipartite graph $B_{5,7}$.

- Identify the sizes of the two vertex sets: $m = 5$ and $n = 7$.
- Use the formula for edges in a complete bipartite graph: $|E| = m \times n$.
- Compute $5 \times 7 = 35$.

Answer: 35

2. A graph G is a hypercube Q_4 . How many vertices does it have, and what is the degree of each vertex?

- The number of vertices in Q_n is 2^n . For $n = 4$, $|V| = 2^4 = 16$.
- The degree of every vertex in Q_n is n . For $n = 4$, $\deg(v) = 4$.

Answer: 16 vertices, degree 4

3. Determine the number of edges in a complete graph K_8 and verify it using the Handshake Lemma.

- Using the combination formula: $|E| = \binom{8}{2} = \frac{8 \times 7}{2} = 28$.
- Using the Handshake Lemma: each of the 8 vertices has degree $8 - 1 = 7$.
- Sum of degrees is $8 \times 7 = 56$.
- Number of edges is $56/2 = 28$.

Answer: 28