

Divide-and-conquer recurrences and the master theorem

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Practice problems

1. Find the growth rate for $T(n) = 4T(n/2) + n$ using the Master Theorem.
2. Solve the recurrence $T(n) = 7T(n/2) + n^2$.
3. Determine the complexity of $T(n) = 2T(n/2) + n \log n$. Does the standard Master Theorem apply?

Solutions

1. Find the growth rate for $T(n) = 4T(n/2) + n$ using the Master Theorem.

- Identify $a = 4$, $b = 2$, and $f(n) = n$.
- Compute the critical value: $n^{\log_2 4} = n^2$.
- Compare $f(n) = n$ with n^2 . Since n is polynomially smaller than n^2 (e.g., $\epsilon = 1$), this is Case 1.
- The growth rate is n^2 .

Answer: n^2

2. Solve the recurrence $T(n) = 7T(n/2) + n^2$.

- Identify $a = 7$, $b = 2$, and $f(n) = n^2$.
- Compute the critical value: $n^{\log_2 7} \approx n^{2.81}$.
- Compare $f(n) = n^2$ with $n^{2.81}$. Since n^2 is polynomially smaller than $n^{2.81}$, this is Case 1.
- The growth rate is $n^{\log_2 7}$.

Answer: $n^{\log_2 7}$

3. Determine the complexity of $T(n) = 2T(n/2) + n \log n$. Does the standard Master Theorem apply?

- Identify $a = 2$, $b = 2$, and $f(n) = n \log n$.
- Compute the critical value: $n^{\log_2 2} = n$.
- Compare $f(n) = n \log n$ with n . $f(n)$ is larger, but not polynomially larger (the difference is only a $\log n$ factor).
- Because there is no $\epsilon > 0$ such that $n \log n$ is at least $n^{1+\epsilon}$, Case 3 does not apply. Case 2 also does not apply because $f(n)$ is not proportional to n .
- The standard Master Theorem does not apply.

Answer: The standard Master Theorem does not apply.