Module 3 Review and Self-Test

Practice problems

1. Calculate the number of ways to partition a set of 6 distinct elements into 2 non-empty identical subsets.
2. Find the number of ways to triangulate a convex polygon with 6 sides.
3. A necklace has 3 beads and 3 available colors. How many distinct necklaces can be made if only rotations are allowed?

Solutions

1. Calculate the number of ways to partition a set of 6 distinct elements into 2 non-empty identical subsets.

   - We need to find the Stirling number of the second kind $S(6, 2)$.
   - The formula for $S(n, 2)$ is $2^{n-1} - 1$.
   - Plugging in $n = 6$, we get $2^{6-1} - 1 = 2^5 - 1$.
   - This equals $32 - 1 = 31$.

   Answer: 31

2. Find the number of ways to triangulate a convex polygon with 6 sides.

   - The number of ways to triangulate a convex polygon with $n + 2$ sides is given by the n-th Catalan number C_n.
   - Here, $n + 2 = 6$, so $n = 4$.
   - We calculate $C_4 = \frac{1}{4+1}\binom{2\times4}{4} = \frac{1}{5}\binom{8}{4}$.
   - $\binom{8}{4} = \frac{8\times7\times6\times5}{4\times3\times2\times1} = 70$.
   - The result is $70/5 = 14$.

   Answer: 14

3. A necklace has 3 beads and 3 available colors. How many distinct necklaces can be made if only rotations are allowed?

   - The group G has 3 rotations: $0°, 120°, 240°$.
   - For $0°$, all $3^3 = 27$ colorings are fixed.
   - For $120°$, all beads must be the same color, so 3 fixed points.
   - For $240°$, all beads must be the same color, so 3 fixed points.
   - By Burnside's Lemma, the number of orbits is $\frac{1}{3}(27 + 3 + 3) = \frac{33}{3} = 11$.

   Answer: 11