Stirling numbers — set partitions and cycles

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Practice problems

1. Compute $S(5, 2)$, the number of ways to partition a set of 5 distinct elements into 2 non-empty subsets.
2. Compute $c(4, 3)$, the number of permutations of 4 elements with exactly 3 disjoint cycles.
3. Find the Bell number B_3 and explain its meaning in terms of equivalence relations.

Solutions

1. Compute $S(5, 2)$, the number of ways to partition a set of 5 distinct elements into 2 non-empty subsets.

   - Use the recurrence $S(n, k) = S(n-1, k-1) + kS(n-1, k)$.
   - For $n = 5, k = 2$: $S(5, 2) = S(4, 1) + 2S(4, 2)$.
   - We know $S(4, 1) = 1$.
   - From the lesson, $S(4, 2) = 7$.
   - Calculate $S(5, 2) = 1 + 2(7) = 1 + 14 = 15$.

   Answer: 15

2. Compute $c(4, 3)$, the number of permutations of 4 elements with exactly 3 disjoint cycles.

   - Use the recurrence $c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k)$.
   - For $n = 4, k = 3$: $c(4, 3) = c(3, 2) + 3c(3, 3)$.
   - From the lesson, $c(3, 2) = 3$.
   - We know $c(3, 3) = 1$.
   - Calculate $c(4, 3) = 3 + 3(1) = 6$.

   Answer: 6

3. Find the Bell number B_3 and explain its meaning in terms of equivalence relations.

   - The Bell number B_3 is the sum $\sum_{k=1}^{3} S(3, k)$.
   - Compute the terms: $S(3, 1) = 1$, $S(3, 2) = 3$, $S(3, 3) = 1$.
   - Sum them: $1 + 3 + 1 = 5$.
   - The meaning is that there are 5 distinct equivalence relations that can be defined on a set of 3 elements.

   Answer: 5
