Stars and bars, deepened; integer compositions

Lesson 38 · Module 3 — Combinatorics, Deepened · Discrete Mathematics · dailymaths.uk/discrete

Practice problems

1. Find the number of strict compositions of $n = 12$ into $k = 4$ parts.
2. Find the number of weak compositions of $n = 8$ into $k = 3$ parts.
3. How many strict compositions of $n = 20$ into $k = 3$ parts exist such that the first part is at least 5?

Solutions

1. Find the number of strict compositions of $n = 12$ into $k = 4$ parts.

   - Identify that this is a strict composition since parts must be positive.
   - Use the formula $\binom{n-1}{k-1}$.
   - Substitute $n = 12$ and $k = 4$ to get $\binom{11}{3}$.
   - Calculate $\frac{11\times10\times9}{3\times2\times1} = 11 \times 5 \times 3 = 165$.

   Answer: 165

2. Find the number of weak compositions of $n = 8$ into $k = 3$ parts.

   - Identify that this is a weak composition since parts can be zero.
   - Use the formula $\binom{n+k-1}{k-1}$.
   - Substitute $n = 8$ and $k = 3$ to get $\binom{8+3-1}{3-1} = \binom{10}{2}$.
   - Calculate $\frac{10\times9}{2\times1} = 45$.

   Answer: 45

3. How many strict compositions of $n = 20$ into $k = 3$ parts exist such that the first part is at least 5?

   - Set constraints: $x_1 \geq 5, x_2 \geq 1, x_3 \geq 1$.
   - Subtract the minimum requirements from the total: $20 - (5 + 1 + 1) = 13$.
   - Distribute the remaining 13 units into 3 parts using the weak formula: $\binom{13+3-1}{3-1}$.
   - Calculate $\binom{15}{2} = \frac{15\times14}{2\times1} = 105$.

   Answer: 105
