Euler's Totient Function and Euler's Theorem

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Practice problems

1. Compute $\phi(72)$.
2. Find the remainder of 11^{42} when divided by 10.
3. Find the modular inverse of 7 modulo 15.

Solutions

1. Compute $\phi(72)$.

   - Find the prime factorization of 72: $72 = 8 \times 9 = 2^3 \times 3^2$.
   - Identify the distinct prime factors: $p_1 = 2$ and $p_2 = 3$.
   - Apply the formula: $\phi(72) = 72(1 - 1/2)(1 - 1/3)$.
   - Calculate: $72 \times (1/2) \times (2/3) = 36 \times (2/3) = 24$.

   Answer: 24

2. Find the remainder of 11^{42} when divided by 10.

   - Check coprimality: $\gcd(11, 10) = 1$.
   - Compute $\phi(10)$: $10 = 2 \times 5$, so $\phi(10) = 10(1/2)(4/5) = 4$.
   - Reduce the exponent modulo $\phi(10)$: $42 \pmod 4 = 2$.
   - Apply Euler's Theorem: $11^{42} \equiv 11^2 \pmod{10}$.
   - Calculate: $11^2 = 121$, and $121 \equiv 1 \pmod{10}$.

   Answer: 1

3. Find the modular inverse of 7 modulo 15.

   - Check coprimality: $\gcd(7, 15) = 1$.
   - Compute $\phi(15)$: $15 = 3 \times 5$, so $\phi(15) = 15(2/3)(4/5) = 8$.
   - The inverse is $7^{\phi(15)-1} = 7^7 \pmod{15}$.
   - Compute $7^7 \pmod{15}$: $7^2 = 49 \equiv 4 \pmod{15}$.
   - Then $7^4 \equiv 4^2 = 16 \equiv 1 \pmod{15}$.
   - Then $7^7 = 7^4 \times 7^2 \times 7^1 \equiv 1 \times 4 \times 7 = 28 \equiv 13 \pmod{15}$.

   Answer: 13
