Modular arithmetic I — clock mathematics

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Practice problems

1. Find the least non negative residue of $17^2 + 11 \cdot 5 \pmod 3$.
2. Compute $3^{10} \pmod 7$.
3. Compute $123 \cdot 456 \pmod 7$.

Solutions

1. Find the least non negative residue of $17^2 + 11 \cdot 5 \pmod{3}$.

   - Simplify the bases first: $17 \equiv 2 \equiv -1 \pmod{3}$, $11 \equiv 2 \equiv -1 \pmod{3}$, and $5 \equiv 2 \equiv -1 \pmod{3}$.
   - Substitute these into the expression: $(-1)^2 + (-1) \cdot (-1) \pmod{3}$.
   - Calculate the result: $1 + 1 = 2$.
   - The residue is 2.

   Answer: 2

2. Compute $3^{10} \pmod{7}$.

   - Use the property of powers: $3^2 = 9 \equiv 2 \pmod{7}$.
   - Then $3^4 = (3^2)^2 \equiv 2^2 = 4 \pmod{7}$.
   - Then $3^8 = (3^4)^2 \equiv 4^2 = 16 \equiv 2 \pmod{7}$.
   - Finally, $3^{10} = 3^8 \cdot 3^2 \equiv 2 \cdot 2 = 4 \pmod{7}$.

   Answer: 4

3. Compute $123 \cdot 456 \pmod{7}$.

   - Simplify each number modulo 7 first.
   - $123 = 7 \cdot 17 + 4$, so $123 \equiv 4 \pmod{7}$.
   - $456 = 7 \cdot 65 + 1$, so $456 \equiv 1 \pmod{7}$.
   - Multiply the residues: $4 \cdot 1 = 4$.
   - The result is 4.

   Answer: 4
