Module 1 review + self-test

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Practice problems

1. Five distinct books are arranged on a shelf. How many ways can they be arranged such that two specific books, A and B, are not adjacent?
2. Let $S = \{1, 2, 3, 4, 5, 6\}$. Define a relation R on S such that aRb if a divides b. Is this an equivalence relation or a partial order? Justify your answer.
3. How many ways can 6 identical balls be placed into 3 distinct boxes such that no box is empty?

Solutions

1. Five distinct books are arranged on a shelf. How many ways can they be arranged such that two specific books, A and B, are not adjacent?

    - Total arrangements of 5 books is $5! = 120$.
    - Treat books A and B as a single block. There are now 4 items to arrange: $(AB), C, D, E$.
    - Ways to arrange these 4 items is $4! = 24$.
    - Within the block, A and B can be arranged in $2! = 2$ ways: (AB) or (BA).
    - Total arrangements where A and B are adjacent is $24 \times 2 = 48$.
    - Subtract adjacent cases from total: $120 - 48 = 72$.

    Answer: 72

2. Let $S = \{1, 2, 3, 4, 5, 6\}$. Define a relation R on S such that aRb if a divides b. Is this an equivalence relation or a partial order? Justify your answer.

    - Reflexivity: a divides a for all $a \in S$. (True)
    - Antisymmetry: If a divides b and b divides a, then $a = b$ for positive integers. (True)
    - Transitivity: If a divides b and b divides c, then a divides c. (True)
    - Since it is reflexive, antisymmetric, and transitive, it is a partial order.

    Answer: Partial order

3. How many ways can 6 identical balls be placed into 3 distinct boxes such that no box is empty?

    - This is a stars and bars problem with the constraint that each box must have at least one ball.
    - We use the formula $\binom{n-1}{k-1}$ where $n = 6$ and $k = 3$.
    - Calculation: $\binom{6-1}{3-1} = \binom{5}{2}$.
    - $\binom{5}{2} = \frac{5\times4}{2\times1} = 10$.

    Answer: 10
