Derangements revisited and the nearest-integer formula

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Practice problems

1. Calculate the number of derangements for $n = 7$.
2. A teacher collects 5 essays and returns them randomly. In how many ways can the teacher return the essays such that exactly 2 students receive their own essay?
3. Use the summation formula $D(n) = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$ to calculate $D(4)$.

Solutions

1. Calculate the number of derangements for $n = 7$.

    - Step 1: Compute $7! = 5040$.
    - Step 2: Use the nearest integer formula $D(7) \approx 5040/e$.
    - Step 3: $5040/2.7182818 \approx 1854.11$.
    - Step 4: Round to the nearest integer.

    Answer: 1854

2. A teacher collects 5 essays and returns them randomly. In how many ways can the teacher return the essays such that exactly 2 students receive their own essay?

    - Step 1: Choose which 2 students get their own essays: $\binom{5}{2} = 10$.
    - Step 2: The remaining 3 students must receive a derangement of their essays: $D(3) = 2$.
    - Step 3: Multiply the choices: $10 \times 2 = 20$.

    Answer: 20

3. Use the summation formula $D(n) = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$ to calculate $D(4)$.

    - Step 1: $D(4) = 4! \left(\frac{1}{0!} - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$.
    - Step 2: $D(4) = 24 \left(1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)$.
    - Step 3: $D(4) = 12 - 4 + 1 = 9$.

    Answer: 9
