

The inclusion-exclusion principle — proved properly

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Practice problems

1. How many integers between 1 and 500 inclusive are divisible by 3, 7, or 11?
2. In a group of 100 students, 60 study Spanish, 40 study French, and 30 study German. 20 study Spanish and French, 15 study Spanish and German, and 10 study French and German. 5 study all three. How many students study none of these languages?
3. Suppose we have 5 sets $A_1, \dots, A_5$. Each set has 20 elements. Every intersection of 2 sets has 5 elements, every intersection of 3 sets has 2 elements, every intersection of 4 sets has 1 element, and the intersection of all 5 sets is empty. Find the size of the union.

Solutions

1. How many integers between 1 and 500 inclusive are divisible by 3, 7, or 11?

- Calculate individual sets: $|A| = \lfloor 500/3 \rfloor = 166$, $|B| = \lfloor 500/7 \rfloor = 71$, $|C| = \lfloor 500/11 \rfloor = 45$.
- Calculate pairs: $|A \cap B| = \lfloor 500/21 \rfloor = 23$, $|A \cap C| = \lfloor 500/33 \rfloor = 15$, $|B \cap C| = \lfloor 500/77 \rfloor = 6$.
- Calculate triple: $|A \cap B \cap C| = \lfloor 500/231 \rfloor = 2$.
- Apply formula: $(166 + 71 + 45) - (23 + 15 + 6) + 2 = 282 - 44 + 2 = 240$.

Answer: 240

2. In a group of 100 students, 60 study Spanish, 40 study French, and 30 study German. 20 study Spanish and French, 15 study Spanish and German, and 10 study French and German. 5 study all three. How many students study none of these languages?

- Find the union: $|S \cup F \cup G| = (60 + 40 + 30) - (20 + 15 + 10) + 5$.
- Compute: $130 - 45 + 5 = 90$.
- Subtract from total: $100 - 90 = 10$.

Answer: 10

3. Suppose we have 5 sets $A_1, \dots, A_5$. Each set has 20 elements. Every intersection of 2 sets has 5 elements, every intersection of 3 sets has 2 elements, every intersection of 4 sets has 1 element, and the intersection of all 5 sets is empty. Find the size of the union.

- Singles: $\binom{5}{1} \times 20 = 100$.
- Pairs: $\binom{5}{2} \times 5 = 10 \times 5 = 50$.
- Triples: $\binom{5}{3} \times 2 = 10 \times 2 = 20$.
- Quadruples: $\binom{5}{4} \times 1 = 5 \times 1 = 5$.
- Quintuples: $\binom{5}{5} \times 0 = 0$.
- Union: $100 - 50 + 20 - 5 + 0 = 65$.

Answer: 65