

Cartesian products and relations — the general idea

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Practice problems

1. Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Find the Cartesian product $A \times B$ and the relation $R = \{(a, b) \in A \times B \mid a + b \text{ is even}\}$.
2. Let $A = \{1, 2, 3\}$. Define a relation R on A such that $(a, b) \in R$ if $a \leq b$. List the elements of R and draw the corresponding grid.
3. If $|A| = 5$ and $|B| = 4$, how many distinct relations can be defined from A to B ?

Solutions

1. Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Find the Cartesian product $A \times B$ and the relation $R = \{(a, b) \in A \times B \mid a + b \text{ is even}\}$.

- Step 1: List all pairs in $A \times B$. These are $(1, 3), (1, 4), (2, 3), (2, 4)$.
- Step 2: Check the sum for each pair. $1 + 3 = 4$ (even), $1 + 4 = 5$ (odd), $2 + 3 = 5$ (odd), $2 + 4 = 6$ (even).
- Step 3: Collect the pairs with even sums.

Answer: $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$, $R = \{(1, 3), (2, 4)\}$

2. Let $A = \{1, 2, 3\}$. Define a relation R on A such that $(a, b) \in R$ if $a \leq b$. List the elements of R and draw the corresponding grid.

- Step 1: Check all pairs (a, b) where $a, b \in \{1, 2, 3\}$.
- Step 2: $1 \leq 1, 1 \leq 2, 1 \leq 3$ are all true.
- Step 3: $2 \leq 2, 2 \leq 3$ are true; $2 \leq 1$ is false.
- Step 4: $3 \leq 3$ is true; $3 \leq 1, 3 \leq 2$ are false.

Answer: $R = \{(1, 1), (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)\}$

3. If $|A| = 5$ and $|B| = 4$, how many distinct relations can be defined from A to B ?

- Step 1: Find the size of the Cartesian product $A \times B$. $|A \times B| = 5 \times 4 = 20$.
- Step 2: A relation is any subset of $A \times B$.
- Step 3: The number of subsets of a set with n elements is 2^n .
- Step 4: Substitute $n = 20$.

Answer: $2^{20} = 1,048,576$