Set algebra proved; power sets and Boolean connections

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Practice problems

1. Use element chasing to prove that $A \cap (B \cap C) = (A \cap B) \cap C$.
2. Let $S = \{1, 2, 3, 4\}$. List all elements of the power set $P(S)$ and state its size.
3. Simplify the set expression $(A^c \cup B)^c \cup (A \cap B^c)$ using set algebra.

Solutions

1. Use element chasing to prove that $A \cap (B \cap C) = (A \cap B) \cap C$.

   - Assume $x \in A \cap (B \cap C)$. Then $x \in A$ and $x \in (B \cap C)$.
   - Since $x \in B \cap C$, then $x \in B$ and $x \in C$.
   - Thus $x \in A$, $x \in B$, and $x \in C$. This implies $x \in (A \cap B)$ and $x \in C$, so $x \in (A \cap B) \cap C$.
   - Now assume $x \in (A \cap B) \cap C$. Then $x \in (A \cap B)$ and $x \in C$.
   - Since $x \in A \cap B$, then $x \in A$ and $x \in B$.
   - Thus $x \in A$ and $x \in (B \cap C)$, so $x \in A \cap (B \cap C)$.

   Answer: The sets are equal by double-inclusion.

2. Let $S = \{1, 2, 3, 4\}$. List all elements of the power set $P(S)$ and state its size.

   - The size is $2^4 = 16$.
   - Subsets of size 0: $\emptyset$.
   - Subsets of size 1: $\{1\}, \{2\}, \{3\}, \{4\}$.
   - Subsets of size 2: $\{1, 2\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}$.
   - Subsets of size 3: $\{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}$.
   - Subsets of size 4: $\{1, 2, 3, 4\}$.

   Answer: 16 subsets.

3. Simplify the set expression $(A^c \cup B)^c \cup (A \cap B^c)$ using set algebra.

   - Apply De Morgan's law to $(A^c \cup B)^c$: it becomes $(A^c)^c \cap B^c$.
   - Simplify the double complement: $A \cap B^c$.
   - The expression is now $(A \cap B^c) \cup (A \cap B^c)$.
   - By the idempotent law, $X \cup X = X$.
   - The result is $A \cap B^c$.

   Answer: $A \cap B^c$
