The Pigeonhole Principle I

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Practice problems

1. How many people must be in a room to guarantee that at least two of them were born on the same day of the month (assuming a month has 31 days)?
2. A bag contains red, blue, and green marbles. How many marbles must you draw to ensure that you have at least 4 marbles of the same color?
3. Prove that in any set of 11 distinct integers, there are at least two whose difference is divisible by 10.

Solutions

1. How many people must be in a room to guarantee that at least two of them were born on the same day of the month (assuming a month has 31 days)?

   - Identify the holes as the days of the month, so $n = 31$.
   - We want at least one hole to have 2 items, so we need $k > n$.
   - The smallest integer k such that $k > 31$ is 32.

   Answer: 32

2. A bag contains red, blue, and green marbles. How many marbles must you draw to ensure that you have at least 4 marbles of the same color?

   - Identify the holes as the colors: red, blue, and green, so $n = 3$.
   - We want $\lceil k/n \rceil = 4$.
   - The worst-case scenario is having 3 of each color, which is $3 \times 3 = 9$ marbles.
   - The 10th marble must create a group of 4.

   Answer: 10

3. Prove that in any set of 11 distinct integers, there are at least two whose difference is divisible by 10.

   - Two integers have a difference divisible by 10 if and only if they have the same remainder when divided by 10.
   - The possible remainders modulo 10 are $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.
   - There are $n = 10$ possible remainders (holes).
   - We have $k = 11$ integers (pigeons).
   - By the Pigeonhole Principle, at least two integers must share the same remainder.
   - Therefore, their difference is a multiple of 10.

   Answer: By the Pigeonhole Principle, 11 integers distributed into 10 remainder classes must result in a collision.
