Predicates and quantifiers — review plus nested-quantifier clinic

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Practice problems

1. Translate the following statement into formal logic using quantifiers: 'Every integer has a successor that is also an integer.' Let the domain be the set of integers $\mathbb{Z}$.
2. Let the domain be the set of all people. Let $L(x, y)$ be the predicate 'x loves y'. What is the difference in meaning between $\forall x \exists y L(x, y)$ and $\exists y \forall x L(x, y)$?
3. Negate the following statement: $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 + y^2 = 0$.

Solutions

1. Translate the following statement into formal logic using quantifiers: 'Every integer has a successor that is also an integer.' Let the domain be the set of integers $\mathbb{Z}$.

    - Identify the predicate: $P(n, m)$ is 'm is the successor of n', which can be written as $m = n + 1$.
    - The statement says 'for every integer n', so we start with $\forall n \in \mathbb{Z}$.
    - It then says 'there exists a successor m', so we add $\exists m \in \mathbb{Z}$.
    - Combine these with the predicate.

    Answer: $\forall n \in \mathbb{Z}, \exists m \in \mathbb{Z}, m = n + 1$

2. Let the domain be the set of all people. Let $L(x, y)$ be the predicate 'x loves y'. What is the difference in meaning between $\forall x \exists y L(x, y)$ and $\exists y \forall x L(x, y)$?

    - Analyze $\forall x \exists y L(x, y)$: For every person x, there is some person y whom x loves. This means everyone loves at least one person (but not necessarily the same person).
    - Analyze $\exists y \forall x L(x, y)$: There is one specific person y such that every person x loves that person y. This means there is one person who is loved by everyone.

    Answer: The first means everyone loves someone; the second means there is one person loved by everyone.

3. Negate the following statement: $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}, x^2 + y^2 = 0$.

    - Apply the negation rule to the first quantifier: $\neg \forall x$ becomes $\exists x$.
    - Apply the negation rule to the second quantifier: $\neg \exists y$ becomes $\forall y$.
    - Negate the inner predicate: $x^2 + y^2 = 0$ becomes $x^2 + y^2 \neq 0$.
    - Combine the results.

    Answer: $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}, x^2 + y^2 \neq 0$
