Normal forms; every truth table is a circuit

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Practice problems

1. Given the truth table where $f(0,0) = 0$, $f(0,1) = 1$, $f(1,0) = 1$, and $f(1,1) = 0$, find the Disjunctive Normal Form (DNF) expression.
2. Convert the expression $f = (x \vee y) \wedge (\neg x \vee \neg y)$ into its DNF equivalent using boolean algebra or a truth table.
3. Show how to implement the OR operation $x \vee y$ using only NAND gates. Provide the sequence of NAND operations.

Solutions

1. Given the truth table where $f(0,0) = 0$, $f(0,1) = 1$, $f(1,0) = 1$, and $f(1,1) = 0$, find the Disjunctive Normal Form (DNF) expression.

   - Identify rows where $f = 1$: these are $(0,1)$ and $(1,0)$.
   - Create minterm for $(0,1)$: since $x = 0$ and $y = 1$, the minterm is $\neg x \land y$.
   - Create minterm for $(1,0)$: since $x = 1$ and $y = 0$, the minterm is $x \land \neg y$.
   - Combine with OR: $f = (\neg x \land y) \lor (x \land \neg y)$.

   Answer: f = (\neg x \land y) \lor (x \land \neg y)

2. Convert the expression $f = (x \lor y) \land (\neg x \lor \neg y)$ into its DNF equivalent using boolean algebra or a truth table.

   - Construct truth table: $f(0,0) = 0$ (because $x \lor y$ is 0), $f(0,1) = 1$, $f(1,0) = 1$, $f(1,1) = 0$ (because $\neg x \lor \neg y$ is 0).
   - Identify true rows: $(0,1)$ and $(1,0)$.
   - Write minterms: $\neg x \land y$ and $x \land \neg y$.
   - Combine: $f = (\neg x \land y) \lor (x \land \neg y)$.

   Answer: f = (\neg x \land y) \lor (x \land \neg y)

3. Show how to implement the OR operation $x \lor y$ using only NAND gates. Provide the sequence of NAND operations.

   - Recall De Morgan's law: $x \lor y = \neg(\neg x \land \neg y)$.
   - Step 1: Compute $\neg x$ using $\text{NAND}(x, x)$.
   - Step 2: Compute $\neg y$ using $\text{NAND}(y, y)$.
   - Step 3: Compute the NAND of these two results: $\text{NAND}(\neg x, \neg y)$.
   - This result is $\neg(\neg x \land \neg y)$, which is $x \lor y$.

   Answer: \textNAND(\textNAND(x, x), \textNAND(y, y))
