Boolean algebra and logic circuits — logic you can build

Practice problems

1. A circuit has two inputs p and q. The inputs are fed into a NAND gate, and the output of that gate is then fed into a NOT gate. Write the final boolean expression and simplify it.

2. Construct the boolean expression for a circuit that outputs 1 if and only if exactly one of the two inputs p and q is 1 (the XOR function) using only AND, OR, and NOT gates.

3. A security system triggers an alarm ($A = 1$) if the master switch ($S = 1$) is on AND (either the door sensor ($D = 1$) is triggered OR the window sensor ($W = 1$) is triggered). Write the boolean expression and determine the output if $S = 1, D = 0, W = 1$.

Solutions

1. A circuit has two inputs p and q. The inputs are fed into a NAND gate, and the output of that gate is then fed into a NOT gate. Write the final boolean expression and simplify it.

   - The NAND gate output is $\overline{p \cdot q}$.
   - The NOT gate negates this output: $\overline{(\overline{p \cdot q})}$.
   - Using the double negation law, $\overline{\overline{A}} = A$.
   - The expression simplifies to $p \cdot q$.

   Answer: $p \cdot q$

2. Construct the boolean expression for a circuit that outputs 1 if and only if exactly one of the two inputs p and q is 1 (the XOR function) using only AND, OR, and NOT gates.

   - Exactly one is 1 means: (p is 1 AND q is 0) OR (p is 0 AND q is 1).
   - Translate this to boolean algebra: $(p \cdot \overline{q}) + (\overline{p} \cdot q)$.
   - This uses two NOT gates, two AND gates, and one OR gate.

   Answer: $(p \cdot \overline{q}) + (\overline{p} \cdot q)$

3. A security system triggers an alarm ($A = 1$) if the master switch ($S = 1$) is on AND (either the door sensor ($D = 1$) is triggered OR the window sensor ($W = 1$) is triggered). Write the boolean expression and determine the output if $S = 1, D = 0, W = 1$.

   - The condition 'either door or window' is $D + W$.
   - The master switch condition is $S \cdot (D + W)$.
   - Substitute the values: $1 \cdot (0 + 1)$.
   - Evaluate the parentheses: $1 \cdot 1 = 1$.

   Answer: $A = S \cdot (D + W)$; Output is 1
