Logical equivalence; the algebra of propositions

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Practice problems

1. Simplify the expression $\neg(p \to q)$ using logical equivalences.
2. Prove that $(p \wedge q) \to p$ is a tautology using a truth table.
3. Simplify the expression $\neg(\neg p \wedge q) \wedge (p \vee q)$.

Solutions

1. Simplify the expression $\neg(p \to q)$ using logical equivalences.

    - Step 1: Use the implication law to rewrite $p \to q$ as $\neg p \vee q$.
    - Step 2: The expression becomes $\neg(\neg p \vee q)$.
    - Step 3: Apply De Morgan's Law to distribute the negation, changing $\vee$ to $\wedge$.
    - Step 4: This gives $\neg(\neg p) \wedge \neg q$.
    - Step 5: Use Double Negation to simplify $\neg(\neg p)$ to p.

    Answer: $p \wedge \neg q$

2. Prove that $(p \wedge q) \to p$ is a tautology using a truth table.

    - Step 1: Create columns for p and q.
    - Step 2: Create a column for $p \wedge q$.
    - Step 3: Evaluate the implication $(p \wedge q) \to p$.
    - Step 4: Observe that in all four rows (TT, TF, FT, FF), the result is True.

    Answer: Tautology

3. Simplify the expression $\neg(\neg p \wedge q) \wedge (p \vee q)$.

    - Step 1: Apply De Morgan's to $\neg(\neg p \wedge q)$, resulting in $\neg(\neg p) \vee \neg q$.
    - Step 2: Simplify to $p \vee \neg q$.
    - Step 3: The expression is now $(p \vee \neg q) \wedge (p \vee q)$.
    - Step 4: Use the Distributive Law in reverse to factor out $p\vee$.
    - Step 5: This gives $p \vee (\neg q \wedge q)$.
    - Step 6: Since $\neg q \wedge q$ is always False, we have $p \vee$ False.
    - Step 7: By Identity Law, this simplifies to p.

    Answer: p
