Propositional logic and truth tables — rapid review, then deepened

Practice problems

1. Construct a truth table for the expression $(p \vee q) \to (p \wedge q)$. Is this expression a tautology, a contradiction, or a contingency?
2. Prove using a truth table that $\neg(p \to q)$ is logically equivalent to $p \wedge \neg q$.
3. Determine if the expression $(p \wedge (p \to q)) \to q$ is a tautology.

Solutions

1. Construct a truth table for the expression $(p \vee q) \to (p \wedge q)$. Is this expression a tautology, a contradiction, or a contingency?

   - Step 1: List combinations of p and q: $(T,T), (T,F), (F,T), (F,F)$.
   - Step 2: Evaluate $p \vee q$: T, T, T, F.
   - Step 3: Evaluate $p \wedge q$: T, F, F, F.
   - Step 4: Evaluate the implication $(p \vee q) \to (p \wedge q)$.
   - Row 1: $T \to T$ is T. Row 2: $T \to F$ is F. Row 3: $T \to F$ is F. Row 4: $F \to F$ is T.

   Answer: The result is T, F, F, T, so it is a contingency.

2. Prove using a truth table that $\neg(p \to q)$ is logically equivalent to $p \wedge \neg q$.

   - Step 1: Evaluate $p \to q$: T, F, T, T.
   - Step 2: Negate the result $\neg(p \to q)$: F, T, F, F.
   - Step 3: Evaluate $\neg q$: F, T, F, T.
   - Step 4: Evaluate $p \wedge \neg q$: $T \wedge F = F$, $T \wedge T = T$, $F \wedge F = F$, $F \wedge T = F$.
   - Step 5: Compare the columns from Step 2 and Step 4.

   Answer: Both columns are F, T, F, F, so they are logically equivalent.

3. Determine if the expression $(p \wedge (p \to q)) \to q$ is a tautology.

   - Step 1: Evaluate $p \to q$: T, F, T, T.
   - Step 2: Evaluate $p \wedge (p \to q)$: $T \wedge T = T$, $T \wedge F = F$, $F \wedge T = F$, $F \wedge T = F$.
   - Step 3: Evaluate the final implication $(p \wedge (p \to q)) \to q$.
   - Row 1: $T \to T$ is T. Row 2: $F \to F$ is T. Row 3: $F \to T$ is T. Row 4: $F \to F$ is T.

   Answer: The result is T, T, T, T, so it is a tautology.
