

Integration by Parts and Substitution — Justified

Lesson 85 · Module 7 — The Riemann Integral · Real Analysis · dailymaths.uk/analysis

Practice problems

1. Evaluate the definite integral $\int_0^{\pi/2} x \cos(x) \, dx$ using integration by parts.
2. Evaluate $\int_0^1 x^2 e^{x^3} \, dx$ using u-substitution.
3. Evaluate $\int_1^e \ln(x) \, dx$ using integration by parts.

Solutions

1. Evaluate the definite integral $\int_0^{\pi/2} x \cos(x) dx$ using integration by parts.

- Let $u = x$, then $du = dx$.
- Let $dv = \cos(x)dx$, then $v = \sin(x)$.
- Apply the formula: $[x \sin(x)]_0^{\pi/2} - \int_0^{\pi/2} \sin(x) dx$.
- Evaluate the boundary term: $(\pi/2) \sin(\pi/2) - 0 = \pi/2$.
- Evaluate the integral: $[-\cos(x)]_0^{\pi/2} = -\cos(\pi/2) - (-\cos(0)) = 0 + 1 = 1$.
- Combine results: $\pi/2 - 1$.

Answer: $\frac{\pi}{2} - 1$

2. Evaluate $\int_0^1 x^2 e^{x^3} dx$ using u-substitution.

- Let $u = x^3$, then $du = 3x^2 dx$, so $x^2 dx = \frac{1}{3} du$.
- Change limits: $x = 0 \implies u = 0$; $x = 1 \implies u = 1$.
- Rewrite integral: $\int_0^1 \frac{1}{3} e^u du$.
- Integrate: $[\frac{1}{3} e^u]_0^1 = \frac{1}{3}(e^1 - e^0)$.
- Simplify: $\frac{1}{3}(e - 1)$.

Answer: $\frac{1}{3}(e - 1)$

3. Evaluate $\int_1^e \ln(x) dx$ using integration by parts.

- Let $u = \ln(x)$, then $du = \frac{1}{x} dx$.
- Let $dv = dx$, then $v = x$.
- Apply formula: $[x \ln(x)]_1^e - \int_1^e x \cdot \frac{1}{x} dx$.
- Boundary term: $e \ln(e) - 1 \ln(1) = e(1) - 0 = e$.
- Integral term: $\int_1^e 1 dx = [x]_1^e = e - 1$.
- Combine: $e - (e - 1) = 1$.

Answer: 1