

The Fundamental Theorem of Calculus II — proved

Lesson 84 · Module 7 — The Riemann Integral · Real Analysis · dailymaths.uk/analysis

Practice problems

1. Evaluate the definite integral $\int_0^2 (3x^2 - 2x + 5) dx$.
2. Evaluate $\int_0^\pi \sin(x) dx$.
3. Find the value of $\int_1^2 (4x^3 + \frac{1}{x}) dx$.

Solutions

1. Evaluate the definite integral $\int_0^2 (3x^2 - 2x + 5) dx$.

- Find the antiderivative of $3x^2 - 2x + 5$. Using the power rule, $F(x) = x^3 - x^2 + 5x$.
- Evaluate F at the upper limit: $F(2) = 2^3 - 2^2 + 5(2) = 8 - 4 + 10 = 14$.
- Evaluate F at the lower limit: $F(0) = 0^3 - 0^2 + 5(0) = 0$.
- Subtract the results: $14 - 0 = 14$.

Answer: 14

2. Evaluate $\int_0^\pi \sin(x) dx$.

- Identify the antiderivative of $\sin(x)$, which is $F(x) = -\cos(x)$.
- Evaluate at the upper limit: $F(\pi) = -\cos(\pi) = -(-1) = 1$.
- Evaluate at the lower limit: $F(0) = -\cos(0) = -(1) = -1$.
- Subtract the results: $1 - (-1) = 2$.

Answer: 2

3. Find the value of $\int_1^2 (4x^3 + \frac{1}{x}) dx$.

- Find the antiderivative: $F(x) = x^4 + \ln(x)$.
- Evaluate at the upper limit: $F(2) = 2^4 + \ln(2) = 16 + \ln(2)$.
- Evaluate at the lower limit: $F(1) = 1^4 + \ln(1) = 1 + 0 = 1$.
- Subtract the results: $(16 + \ln(2)) - 1 = 15 + \ln(2)$.

Answer: $15 + \ln(2)$