

# The Fundamental Theorem of Calculus I — proved

Lesson 83 · Module 7 — The Riemann Integral · Real Analysis · [dailymaths.uk/analysis](http://dailymaths.uk/analysis)

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## Practice problems

1. Let  $G(x) = \int_1^{x^2} \cos(t) dt$ . Find  $G'(x)$  using the Fundamental Theorem of Calculus and the chain rule.
2. Suppose  $f$  is continuous on  $[0, 1]$  and  $F(x) = \int_0^x f(t) dt$ . If  $F(x) = x^2$ , what is the function  $f(x)$ ?
3. Prove that if  $f(x) = c$  for some constant  $c$ , then the area function  $F(x) = \int_a^x f(t) dt$  satisfies  $F'(x) = c$ .

## Solutions

1. Let  $G(x) = \int_1^{x^2} \cos(t) dt$ . Find  $G'(x)$  using the Fundamental Theorem of Calculus and the chain rule.

- Let  $u = x^2$ . Then  $G(x) = \int_1^u \cos(t) dt$ .
- By FTC Part 1,  $\frac{dG}{du} = \cos(u)$ .
- By the chain rule,  $G'(x) = \frac{dG}{du} \cdot \frac{du}{dx}$ .
- Substitute  $u = x^2$  and  $\frac{du}{dx} = 2x$ .
- Result is  $\cos(x^2) \cdot 2x$ .

**Answer:**  $2x \cos(x^2)$

2. Suppose  $f$  is continuous on  $[0, 1]$  and  $F(x) = \int_0^x f(t) dt$ . If  $F(x) = x^2$ , what is the function  $f(x)$ ?

- By FTC Part 1,  $F'(x) = f(x)$ .
- We are given  $F(x) = x^2$ .
- Differentiate  $F(x)$  with respect to  $x$ :  $F'(x) = 2x$ .
- Therefore,  $f(x) = 2x$ .

**Answer:**  $f(x) = 2x$

3. Prove that if  $f(x) = c$  for some constant  $c$ , then the area function  $F(x) = \int_a^x f(t) dt$  satisfies  $F'(x) = c$ .

- The integral of a constant  $c$  from  $a$  to  $x$  is  $F(x) = c(x - a)$ .
- Differentiate  $F(x)$  with respect to  $x$ .
- The derivative of  $cx$  is  $c$ , and the derivative of the constant  $-ca$  is 0.
- Thus  $F'(x) = c$ , which matches  $f(x)$ .

**Answer:**  $F'(x) = c$