

Differentiation rules proved

Lesson 68 · Module 6 — Differentiation, Rigorously · Real Analysis · dailymaths.uk/analysis

Practice problems

1. Use the product rule to find the derivative of $f(x) = x^3 \ln(x)$.
2. Find the derivative of $g(x) = \frac{\sin(x)}{x^2}$ using the quotient rule.
3. Find the derivative of $h(x) = e^{\sin(x^2)}$ using the chain rule twice.

Solutions

1. Use the product rule to find the derivative of $f(x) = x^3 \ln(x)$.

- Identify $u = x^3$ and $v = \ln(x)$.
- Compute $u' = 3x^2$ and $v' = 1/x$.
- Apply the product rule: $f'(x) = u'v + uv'$.
- Substitute: $f'(x) = 3x^2 \ln(x) + x^3(1/x)$.
- Simplify: $f'(x) = 3x^2 \ln(x) + x^2$.

Answer: $f'(x) = x^2(3\ln(x) + 1)$

2. Find the derivative of $g(x) = \frac{\sin(x)}{x^2}$ using the quotient rule.

- Identify numerator $u = \sin(x)$ and denominator $v = x^2$.
- Compute $u' = \cos(x)$ and $v' = 2x$.
- Apply the quotient rule: $g'(x) = \frac{u'v - uv'}{v^2}$.
- Substitute: $g'(x) = \frac{\cos(x)x^2 - \sin(x)(2x)}{(x^2)^2}$.
- Simplify: $g'(x) = \frac{x^2 \cos(x) - 2x \sin(x)}{x^4} = \frac{x \cos(x) - 2 \sin(x)}{x^3}$.

Answer: $g'(x) = \frac{\cos(x) - 2\sin(x)}{x^3}$

3. Find the derivative of $h(x) = e^{\sin(x^2)}$ using the chain rule twice.

- Identify the outermost function as e^u , where $u = \sin(x^2)$.
- The derivative of e^u is $e^u \cdot u'$.
- Now find $u' = \frac{d}{dx}[\sin(x^2)]$. This is another chain rule: $\cos(x^2) \cdot (2x)$.
- Combine the results: $h'(x) = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x$.

Answer: $h'(x) = 2x \cos(x^2) e^{\sin(x^2)}$