eps-delta proof clinic I

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Practice problems

1. Prove using the epsilon-delta definition that $\lim_{x\to 3}(4x-5)=7$.
2. Prove using the epsilon-delta definition that $\lim_{x\to 1}(2x^2+1)=3$.
3. Prove using the epsilon-delta definition that $\lim_{x\to 2}(x^2-3x)=-2$.

Solutions

1. Prove using the epsilon-delta definition that $\lim_{x\to 3}(4x-5)=7$.

   - Scratch work: $|(4x-5)-7| = |4x-12| = 4|x-3|$.
   - We want $4|x-3|<\epsilon$, which means $|x-3|<\epsilon/4$.
   - Formal proof: Given $\epsilon>0$, choose $\delta=\epsilon/4$.
   - If $0<|x-3|<\delta$, then $|(4x-5)-7| = 4|x-3| < 4\delta = 4(\epsilon/4)=\epsilon$.

   Answer: \delta = \epsilon/4

2. Prove using the epsilon-delta definition that $\lim_{x\to 1}(2x^2+1)=3$.

   - Scratch work: $|(2x^2+1)-3| = |2x^2-2| = 2|x^2-1| = 2|x-1||x+1|$.
   - Assume $\delta \le 1$. Then $|x-1|<1 \implies 0<x<2 \implies |x+1|<3$.
   - We need $2|x-1|(3)<\epsilon$, so $|x-1|<\epsilon/6$.
   - Choose $\delta=\min(1,\epsilon/6)$.
   - If $0<|x-1|<\delta$, then $|2x^2-2| = 2|x-1||x+1| < 2(\epsilon/6)(3)=\epsilon$.

   Answer: \delta = \min(1, \epsilon/6)

3. Prove using the epsilon-delta definition that $\lim_{x\to 2}(x^2-3x)=-2$.

   - Scratch work: $|(x^2-3x)-(-2)| = |x^2-3x+2| = |x-2||x-1|$.
   - Assume $\delta \le 1$. Then $|x-2|<1 \implies 1<x<3 \implies |x-1|<2$.
   - We need $|x-2|(2)<\epsilon$, so $|x-2|<\epsilon/2$.
   - Choose $\delta=\min(1,\epsilon/2)$.
   - If $0<|x-2|<\delta$, then $|x^2-3x+2| = |x-2||x-1| < (\epsilon/2)(2)=\epsilon$.

   Answer: \delta = \min(1, \epsilon/2)