

Open and closed sets — the new vocabulary of nearness

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Practice problems

1. Determine if the set $S = \mathbb{R} \setminus \{0\}$ is open, closed, neither, or both.
2. Is the set $S = \{x \in \mathbb{R} : x^2 > 1\}$ open or closed?
3. Consider the set $S = \{1, 1/2, 1/3, 1/4, \dots\}$. Is this set open or closed?

Solutions

1. Determine if the set $S = \mathbb{R} \setminus \{0\}$ is open, closed, neither, or both.

- Pick any $x \in S$. Since $x \neq 0$, the distance to the origin is $|x| > 0$.
- Choose $\epsilon = |x|/2$. The interval $(x - \epsilon, x + \epsilon)$ does not contain 0 because the distance from x to 0 is $|x|$, which is greater than ϵ .
- Thus, every point in S is an interior point, so S is open.
- Now check if it is closed. The sequence $x_n = 1/n$ is in S for all n , but its limit is 0.
- Since $0 \notin S$, the set does not contain all its limit points, so it is not closed.

Answer: Open

2. Is the set $S = \{x \in \mathbb{R} : x^2 > 1\}$ open or closed?

- Solve the inequality: $x^2 > 1$ implies $x > 1$ or $x < -1$.
- Pick any $x \in S$. If $x > 1$, let $\epsilon = x - 1$. The interval $(x - \epsilon, x + \epsilon) = (1, 2x - 1)$ is a subset of $(1, \infty)$, which is a subset of S .
- If $x < -1$, let $\epsilon = -1 - x$. The interval $(x - \epsilon, x + \epsilon) = (2x + 1, -1)$ is a subset of $(-\infty, -1)$, which is a subset of S .
- Since every point in S is an interior point, S is open.
- To check if it is closed, consider the sequence $x_n = 1 + 1/n$. This sequence is in S and converges to 1.
- Since $1 \notin S$, the set is not closed.

Answer: Open

3. Consider the set $S = \{1, 1/2, 1/3, 1/4, \dots\}$. Is this set open or closed?

- Check for openness: Pick the point $1 \in S$. Any interval $(1 - \epsilon, 1 + \epsilon)$ contains numbers like $1 + \epsilon/2$, which are not in S . Thus, S is not open.
- Check for closedness: A set is closed if it contains all its limit points.
- The only limit point of the sequence $1/n$ is 0.
- Is 0 in the set S ? No, $1/n$ is never equal to 0 for any natural number n .
- Since the limit point 0 is not in S , the set is not closed.

Answer: Neither