Decimal and b-adic expansions as series

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Practice problems

1. Find the binary expansion (base $b = 2$) of the fraction $\frac{7}{16}$.
2. In base $b = 5$, what is the non-terminating expansion of the number $\frac{1}{5}$?
3. Prove that the series $\sum_{n=1}^{\infty} \frac{2}{3^n}$ converges to 1 using the geometric series formula.

Solutions

1. Find the binary expansion (base $b = 2$) of the fraction $\frac{7}{16}$.

    - Multiply $\frac{7}{16}$ by 2: $\frac{14}{16} = 0.875$, so $d_1 = 0$.
    - Multiply 0.875 by 2: 1.75, so $d_2 = 1$. Remainder is 0.75.
    - Multiply 0.75 by 2: 1.5, so $d_3 = 1$. Remainder is 0.5.
    - Multiply 0.5 by 2: 1.0, so $d_4 = 1$. Remainder is 0.
    - The expansion is 0.0111_2.

    Answer: 0.0111_2

2. In base $b = 5$, what is the non-terminating expansion of the number $\frac{1}{5}$?

    - The terminating expansion is 0.1_5.
    - To find the non-terminating version, subtract 1 from the last non-zero digit: $1 - 1 = 0$.
    - Append an infinite string of the digit $b - 1$, which is $5 - 1 = 4$.
    - The expansion is $0.0444\ldots_5$.

    Answer: 0.0444\dots_5

3. Prove that the series $\sum_{n=1}^{\infty} \frac{2}{3^n}$ converges to 1 using the geometric series formula.

    - Identify the first term $a = \frac{2}{3^1} = \frac{2}{3}$.
    - Identify the common ratio $r = \frac{1}{3}$.
    - Apply the formula $S = \frac{a}{1-r}$.
    - Compute $S = \frac{2/3}{1-1/3} = \frac{2/3}{2/3} = 1$.

    Answer: 1

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