Series as sequences of partial sums; geometric series proved

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Practice problems

1. Compute the sum of the geometric series $\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$.
2. Determine if the series $\sum_{n=0}^{\infty} 3\left(-\frac{2}{3}\right)^{n}$ converges, and if so, find its sum.
3. Find the sum of the series $\sum_{n=2}^{\infty} \left(\frac{1}{3}\right)^{n}$.

Solutions

1. Compute the sum of the geometric series $\sum_{n=1}^{\infty} 5\left(\frac{1}{4}\right)^{n-1}$.

   - Identify the first term a by plugging in $n = 1$: $a = 5(1/4)^0 = 5$.
   - Identify the common ratio $r = 1/4$.
   - Check convergence: $|1/4| < 1$, so the series converges.
   - Apply the formula $S = a/(1-r) = 5/(1-1/4) = 5/(3/4)$.
   - Simplify the fraction: $5 \cdot (4/3) = 20/3$.

   Answer: 20/3

2. Determine if the series $\sum_{n=0}^{\infty} 3\left(-\frac{2}{3}\right)^{n}$ converges, and if so, find its sum.

   - Identify the first term $a = 3(-2/3)^0 = 3$.
   - Identify the common ratio $r = -2/3$.
   - Check convergence: $|-2/3| = 2/3 < 1$, so the series converges.
   - Apply the formula $S = a/(1-r) = 3/(1-(-2/3)) = 3/(5/3)$.
   - Simplify the fraction: $3 \cdot (3/5) = 9/5$.

   Answer: 9/5

3. Find the sum of the series $\sum_{n=2}^{\infty} \left(\frac{1}{3}\right)^{n}$.

   - Identify the first term a by plugging in the starting index $n = 2$: $a = (1/3)^2 = 1/9$.
   - Identify the common ratio $r = 1/3$.
   - Check convergence: $|1/3| < 1$, so the series converges.
   - Apply the formula $S = a/(1-r) = (1/9)/(1-1/3) = (1/9)/(2/3)$.
   - Simplify the fraction: $(1/9) \cdot (3/2) = 3/18 = 1/6$.

   Answer: 1/6
