Proof by contradiction — root 2 is irrational

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Practice problems

1. Use a proof by contradiction to show that if n^2 is an even integer, then n must be an even integer.
2. Prove by contradiction that there is no largest integer.
3. Prove by contradiction that $\sqrt{3}$ is irrational.

Solutions

1. Use a proof by contradiction to show that if n^2 is an even integer, then n must be an even integer.

   - Assume n^2 is even but n is odd.
   - Since n is odd, we can write $n = 2k + 1$ for some integer k.
   - Then $n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$.
   - This shows n^2 is odd, which contradicts the hypothesis that n^2 is even.
   - Therefore, n must be even.

   Answer: n \text is even

2. Prove by contradiction that there is no largest integer.

   - Assume there exists a largest integer M.
   - Consider the number $M + 1$.
   - Since M is an integer, $M + 1$ is also an integer.
   - By definition, $M + 1 > M$.
   - This contradicts the assumption that M is the largest integer.
   - Therefore, no largest integer exists.

   Answer: \textNo largest integer exists

3. Prove by contradiction that $\sqrt{3}$ is irrational.

   - Assume $\sqrt{3} = p/q$ in simplest form.
   - Then $3 = p^2/q^2$, so $3q^2 = p^2$.
   - This implies p^2 is a multiple of 3, so p must be a multiple of 3. Let $p = 3k$.
   - Substitute: $3q^2 = (3k)^2 = 9k^2$, so $q^2 = 3k^2$.
   - This implies q^2 is a multiple of 3, so q must be a multiple of 3.
   - Since both p and q are multiples of 3, the fraction p/q was not in simplest form, a contradiction.

   Answer: \sqrt3 \notin \mathbbQ
