Proof by contrapositive

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Practice problems

1. Prove that for any integer n, if $3n + 2$ is even, then n is even.
2. Prove that for any two integers x and y, if xy is even, then x is even or y is even.
3. Prove that for any integer n, if $n^2 - 2n + 7$ is even, then n is odd.

Solutions

1. Prove that for any integer n, if $3n + 2$ is even, then n is even.

   - State the contrapositive: If n is odd, then $3n + 2$ is odd.
   - Assume n is odd, so $n = 2k + 1$ for some integer k.
   - Substitute into the expression: $3(2k + 1) + 2 = 6k + 3 + 2 = 6k + 5$.
   - Rewrite as $2(3k + 2) + 1$.
   - Since $3k + 2$ is an integer, $3n + 2$ is odd.
   - The contrapositive is true, so the original statement is true.

   Answer: Proven via contrapositive.

2. Prove that for any two integers x and y, if xy is even, then x is even or y is even.

   - State the contrapositive: If x is odd AND y is odd, then xy is odd.
   - Assume $x = 2k + 1$ and $y = 2m + 1$ for integers k, m.
   - Compute the product: $xy = (2k + 1)(2m + 1) = 4km + 2k + 2m + 1$.
   - Factor out a two: $xy = 2(2km + k + m) + 1$.
   - Since $2km + k + m$ is an integer, xy is odd.
   - The contrapositive is true, so the original statement is true.

   Answer: Proven via contrapositive.

3. Prove that for any integer n, if $n^2 - 2n + 7$ is even, then n is odd.

   - State the contrapositive: If n is even, then $n^2 - 2n + 7$ is odd.
   - Assume $n = 2k$ for some integer k.
   - Substitute: $(2k)^2 - 2(2k) + 7 = 4k^2 - 4k + 7$.
   - Rewrite as $4k^2 - 4k + 6 + 1 = 2(2k^2 - 2k + 3) + 1$.
   - Since $2k^2 - 2k + 3$ is an integer, the result is odd.
   - The contrapositive is true, so the original statement is true.

   Answer: Proven via contrapositive.
