

The unified view — all four theorems as one

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Practice problems

1. Compute the flux of $\mathbf{F} = \langle x, y, z \rangle$ through the surface of a cube with vertices at $(0, 0, 0)$ and $(2, 2, 2)$ using the Divergence Theorem.
2. Use the unified view to explain why the line integral of a conservative field $\mathbf{F} = \nabla f$ around a closed loop C is always zero.
3. Find the triple integral of $f(x, y, z) = 3$ over the region V bounded by the sphere $x^2 + y^2 + z^2 = 4$. Then, identify a vector field $\mathbf{F}$ such that this integral represents the total flux through the boundary surface S via the Divergence Theorem.

Solutions

1. Compute the flux of $\mathbf{F} = \langle x, y, z \rangle$ through the surface of a cube with vertices at $(0, 0, 0)$ and $(2, 2, 2)$ using the Divergence Theorem.
 - Compute the divergence: $\nabla \cdot \mathbf{F} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 1 + 1 + 1 = 3$.
 - Identify the volume of the cube: $V = 2 \times 2 \times 2 = 8$.
 - Apply the Divergence Theorem: $\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_V 3 dV = 3 \times 8 = 24$.

Answer: 24

2. Use the unified view to explain why the line integral of a conservative field $\mathbf{F} = \nabla f$ around a closed loop C is always zero.
 - By the unified view (Stokes' Theorem), the line integral around C equals the surface integral of the curl of $\mathbf{F}$ over a surface S bounded by C .
 - The curl of a gradient field is always zero: $\nabla \times (\nabla f) = 0$.
 - Integrating zero over the surface S results in zero.

Answer: 0

3. Find the triple integral of $f(x, y, z) = 3$ over the region V bounded by the sphere $x^2 + y^2 + z^2 = 4$. Then, identify a vector field $\mathbf{F}$ such that this integral represents the total flux through the boundary surface S via the Divergence Theorem.
 - The integral is $3 \times \text{Volume of sphere} = 3 \times (\frac{4}{3}\pi 2^3) = 3 \times \frac{32\pi}{3} = 32\pi$.
 - We need $\nabla \cdot \mathbf{F} = 3$.
 - A simple field satisfying this is $\mathbf{F} = \langle x, y, z \rangle$.

Answer: 32π , $\mathbf{F} = \langle x, y, z \rangle$