

The Divergence Theorem I — statement and intuition

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Practice problems

1. Compute the flux of $\mathbf{F} = \langle x, 2y, 3z \rangle$ through the sphere $x^2 + y^2 + z^2 = 1$.
2. Find the flux of $\mathbf{F} = \langle x^3, y^3, z^3 \rangle$ through the unit cube $[0, 1] \times [0, 1] \times [0, 1]$.
3. A vector field has a constant divergence $\nabla \cdot \mathbf{F} = -5$. If the flux through a closed surface S is -100π , what is the volume of the region enclosed by S ?

Solutions

1. Compute the flux of $\mathbf{F} = \langle x, 2y, 3z \rangle$ through the sphere $x^2 + y^2 + z^2 = 1$.

- Compute the divergence: $\nabla \cdot \mathbf{F} = 1 + 2 + 3 = 6$.
- Apply the Divergence Theorem: $\Phi = \iiint_V 6dV$.
- The volume of the sphere is $\frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$.
- The flux is $6 \cdot \frac{4}{3}\pi = 8\pi$.

Answer: 8π

2. Find the flux of $\mathbf{F} = \langle x^3, y^3, z^3 \rangle$ through the unit cube $[0, 1] \times [0, 1] \times [0, 1]$.

- Compute the divergence: $\nabla \cdot \mathbf{F} = 3x^2 + 3y^2 + 3z^2$.
- Set up the triple integral: $\int_0^1 \int_0^1 \int_0^1 (3x^2 + 3y^2 + 3z^2) dx dy dz$.
- Integrate $3x^2$ over the cube: $\int_0^1 3x^2 dx = [x^3]_0^1 = 1$.
- By symmetry, the integrals of $3y^2$ and $3z^2$ are also 1.
- Total flux is $1 + 1 + 1 = 3$.

Answer: 3

3. A vector field has a constant divergence $\nabla \cdot \mathbf{F} = -5$. If the flux through a closed surface S is -100π , what is the volume of the region enclosed by S ?

- Use the Divergence Theorem: $\Phi = \iiint_V (\nabla \cdot \mathbf{F}) dV$.
- Substitute the known values: $-100\pi = \iiint_V -5 dV$.
- Pull the constant out: $-100\pi = -5 \cdot \text{Volume}(V)$.
- Solve for volume: $\text{Volume}(V) = \frac{-100\pi}{-5} = 20\pi$.

Answer: 20π