

Flux form of Green's theorem — the 2D divergence theorem

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Practice problems

1. Compute the outward flux of the vector field $\mathbf{F} = 3x\mathbf{i} + 2y\mathbf{j}$ across the boundary of the disk $x^2 + y^2 \leq 9$.
2. Find the net outward flux of $\mathbf{F} = (x + y)\mathbf{i} + (x - y)\mathbf{j}$ across the boundary of the rectangle $0 \leq x \leq 2, 0 \leq y \leq 1$.
3. Calculate the outward flux of $\mathbf{F} = e^y\mathbf{i} + x^2\mathbf{j}$ across the boundary of the region R defined by $0 \leq x \leq 1$ and $0 \leq y \leq x$.

Solutions

1. Compute the outward flux of the vector field $\mathbf{F} = 3x\mathbf{i} + 2y\mathbf{j}$ across the boundary of the disk $x^2 + y^2 \leq 9$.
 - Identify $P = 3x$ and $Q = 2y$.
 - Compute the divergence: $\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} = 3 + 2 = 5$.
 - Apply the flux form of Green's Theorem: $\text{Flux} = \iint_R 5dA$.
 - The area of the disk with radius 3 is $\pi(3)^2 = 9\pi$.
 - The total flux is $5 \times 9\pi = 45\pi$.

Answer: 45π

2. Find the net outward flux of $\mathbf{F} = (x + y)\mathbf{i} + (x - y)\mathbf{j}$ across the boundary of the rectangle $0 \leq x \leq 2, 0 \leq y \leq 1$.
 - Identify $P = x + y$ and $Q = x - y$.
 - Compute the divergence: $\frac{\partial P}{\partial x} = 1$ and $\frac{\partial Q}{\partial y} = -1$.
 - The divergence is $1 + (-1) = 0$.
 - The integral of zero over any region is zero.

Answer: 0

3. Calculate the outward flux of $\mathbf{F} = e^y\mathbf{i} + x^2\mathbf{j}$ across the boundary of the region R defined by $0 \leq x \leq 1$ and $0 \leq y \leq x$.
 - Identify $P = e^y$ and $Q = x^2$.
 - Compute the divergence: $\frac{\partial P}{\partial x} = 0$ and $\frac{\partial Q}{\partial y} = 0$.
 - The divergence is $0 + 0 = 0$.
 - The total flux is the double integral of zero, which is zero.

Answer: 0