Curl — local rotation

Lesson 40 · Module 3 — Vector Fields & Line Integrals · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

Practice problems

1. Compute the curl of the vector field $\mathbf{F}(x, y) = 3xy\mathbf{i} + x^2\mathbf{j}$. Is the field irrotational?
2. Find the curl of $\mathbf{F}(x, y) = e^x\mathbf{i} + \sin(y)\mathbf{j}$.
3. A vector field is given by $\mathbf{F}(x, y) = (y^3)\mathbf{i} + (3xy^2)\mathbf{j}$. Determine the curl and explain the rotation at the point $(1, 2)$.

Solutions

1. Compute the curl of the vector field $\mathbf{F}(x, y) = 3xy\mathbf{i} + x^2\mathbf{j}$. Is the field irrotational?

   - Identify $P = 3xy$ and $Q = x^2$.
   - Compute $\frac{\partial Q}{\partial x} = 2x$.
   - Compute $\frac{\partial P}{\partial y} = 3x$.
   - Subtract them: curl $\mathbf{F} = 2x - 3x = -x$.

   Answer: \textcurl \mathbfF = -x; \text not irrotational

2. Find the curl of $\mathbf{F}(x, y) = e^x\mathbf{i} + \sin(y)\mathbf{j}$.

   - Identify $P = e^x$ and $Q = \sin(y)$.
   - Compute $\frac{\partial Q}{\partial x} = 0$ since $\sin(y)$ does not depend on x.
   - Compute $\frac{\partial P}{\partial y} = 0$ since e^x does not depend on y.
   - Subtract them: $0 - 0 = 0$.

   Answer: \textcurl \mathbfF = 0

3. A vector field is given by $\mathbf{F}(x, y) = (y^3)\mathbf{i} + (3xy^2)\mathbf{j}$. Determine the curl and explain the rotation at the point $(1, 2)$.

   - Identify $P = y^3$ and $Q = 3xy^2$.
   - Compute $\frac{\partial Q}{\partial x} = 3y^2$.
   - Compute $\frac{\partial P}{\partial y} = 3y^2$.
   - Subtract them: $3y^2 - 3y^2 = 0$.

   Answer: \textcurl \mathbfF = 0; \text no rotation at (1, 2)

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