Divergence — local sources and sinks

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Practice problems

1. Compute the divergence of the vector field $\mathbf{F}(x, y) = \langle x^3 + y^2, \cos(xy)\rangle$.
2. Determine if the vector field $\mathbf{F}(x, y, z) = \langle yz, xz, xy\rangle$ is solenoidal.
3. Find the divergence of $\mathbf{F}(x, y) = \langle \ln(x^2 + 1), e^{2y}\rangle$ and evaluate it at the point $(1, 0)$.

Solutions

1. Compute the divergence of the vector field $\mathbf{F}(x, y) = \langle x^3 + y^2, \cos(xy) \rangle$.

   - Identify $P(x, y) = x^3 + y^2$ and $Q(x, y) = \cos(xy)$.
   - Compute $\frac{\partial P}{\partial x} = 3x^2$.
   - Compute $\frac{\partial Q}{\partial y} = -x\sin(xy)$ using the chain rule.
   - Sum the partials: div $\mathbf{F} = 3x^2 - x\sin(xy)$.

   Answer: 3xˆ2 - x\sin(xy)

2. Determine if the vector field $\mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$ is solenoidal.

   - Identify components: $P = yz$, $Q = xz$, $R = xy$.
   - Compute $\frac{\partial P}{\partial x} = 0$.
   - Compute $\frac{\partial Q}{\partial y} = 0$.
   - Compute $\frac{\partial R}{\partial z} = 0$.
   - Sum the partials: $0 + 0 + 0 = 0$.

   Answer: Yes, it is solenoidal.

3. Find the divergence of $\mathbf{F}(x, y) = \langle \ln(x^2 + 1), e^{2y} \rangle$ and evaluate it at the point $(1, 0)$.

   - Compute $\frac{\partial P}{\partial x} = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$.
   - Compute $\frac{\partial Q}{\partial y} = 2e^{2y}$.
   - The general divergence is div $\mathbf{F} = \frac{2x}{x^2+1} + 2e^{2y}$.
   - Evaluate at $(1, 0)$: $\frac{2(1)}{1^2+1} + 2e^{2(0)} = \frac{2}{2} + 2(1) = 1 + 2 = 3$.

   Answer: 3
