When is a field conservative? The curl test in 2D

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Practice problems

1. Determine if the vector field $\mathbf{F}(x, y) = \langle 3x^2 + y, 2xy + e^y \rangle$ is conservative on the entire xy-plane.
2. Check if $\mathbf{F}(x, y) = \langle 2x + e^y, xe^y + 3y^2 \rangle$ is conservative on the entire xy-plane.
3. Consider the field $\mathbf{F}(x, y) = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$. Is it conservative on its domain $\mathbb{R}^2 \setminus \{(0, 0)\}$?

Solutions

1. Determine if the vector field $\mathbf{F}(x, y) = \langle 3x^2 + y, 2xy + e^y \rangle$ is conservative on the entire xy-plane.

   - Identify $P = 3x^2 + y$ and $Q = 2xy + e^y$.
   - Compute $\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}(3x^2 + y) = 1$.
   - Compute $\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}(2xy + e^y) = 2y$.
   - Compare the results: $1 \neq 2y$.

   Answer: Not conservative

2. Check if $\mathbf{F}(x, y) = \langle 2x + e^y, xe^y + 3y^2 \rangle$ is conservative on the entire xy-plane.

   - Identify $P = 2x + e^y$ and $Q = xe^y + 3y^2$.
   - Compute $\frac{\partial P}{\partial y} = e^y$.
   - Compute $\frac{\partial Q}{\partial x} = e^y$.
   - Since $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ and the domain is simply connected, the field is conservative.

   Answer: Conservative

3. Consider the field $\mathbf{F}(x, y) = \langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \rangle$. Is it conservative on its domain $\mathbb{R}^2 \setminus \{(0, 0)\}$?

   - Identify $P = \frac{-y}{x^2+y^2}$ and $Q = \frac{x}{x^2+y^2}$.
   - Compute $\frac{\partial P}{\partial y} = \frac{y^2-x^2}{(x^2+y^2)^2}$ using the quotient rule.
   - Compute $\frac{\partial Q}{\partial x} = \frac{y^2-x^2}{(x^2+y^2)^2}$ using the quotient rule.
   - The partials are equal, but the domain is not simply connected. A line integral around the unit circle yields 2π, not 0, so the field is not conservative.

   Answer: Not conservative
