Vector fields — wind maps, force fields, flows

Lesson 33 · Module 3 — Vector Fields & Line Integrals · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

Practice problems

1. Given the vector field $\mathbf{F}(x, y) = (x + y)\mathbf{i} + (x^2 y)\mathbf{j}$, find the vector at the point $(1, -2)$.
2. Find the gradient vector field ∇f for the scalar function $f(x, y) = e^{xy}$. Then, evaluate the vector at the point $(0, 1)$.
3. A 3D vector field is given by $\mathbf{F}(x, y, z) = \frac{x}{x^2+y^2+z^2}\mathbf{i} + \frac{y}{x^2+y^2+z^2}\mathbf{j} + \frac{z}{x^2+y^2+z^2}\mathbf{k}$. Describe the direction of the vectors in this field and find the vector at $(1, 1, 1)$.

Solutions

1. Given the vector field $\mathbf{F}(x, y) = (x + y)\mathbf{i} + (x^2y)\mathbf{j}$, find the vector at the point $(1, -2)$.

   - Identify $P(x, y) = x + y$ and $Q(x, y) = x^2y$.
   - Substitute $x = 1$ and $y = -2$ into P: $P(1, -2) = 1 + (-2) = -1$.
   - Substitute $x = 1$ and $y = -2$ into Q: $Q(1, -2) = (1)^2(-2) = -2$.
   - Combine the components into a vector.

   Answer: \mathbfF(1, -2) = -1\mathbfi - 2\mathbfj \text or \langle -1, -2 \rangle

2. Find the gradient vector field ∇f for the scalar function $f(x, y) = e^{xy}$. Then, evaluate the vector at the point $(0, 1)$.

   - Compute the partial derivative with respect to x: $\frac{\partial f}{\partial x} = ye^{xy}$.
   - Compute the partial derivative with respect to y: $\frac{\partial f}{\partial y} = xe^{xy}$.
   - Write the gradient field: $\nabla f = ye^{xy}\mathbf{i} + xe^{xy}\mathbf{j}$.
   - Substitute $x = 0$ and $y = 1$: $\nabla f(0, 1) = (1)e^0\mathbf{i} + (0)e^0\mathbf{j} = 1\mathbf{i} + 0\mathbf{j}$.

   Answer: \nabla f(0, 1) = \mathbfi \text or \langle 1, 0 \rangle

3. A 3D vector field is given by $\mathbf{F}(x, y, z) = \frac{x}{x^2+y^2+z^2}\mathbf{i} + \frac{y}{x^2+y^2+z^2}\mathbf{j} + \frac{z}{x^2+y^2+z^2}\mathbf{k}$. Describe the direction of the vectors in this field and find the vector at $(1, 1, 1)$.

   - Observe that the vector is a scalar multiple of the position vector $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.
   - The scalar multiple is $1/(x^2 + y^2 + z^2)$, which is always positive. Thus, all vectors point radially away from the origin.
   - Substitute $x = 1, y = 1, z = 1$ into the components.
   - The denominator is $1^2 + 1^2 + 1^2 = 3$.
   - The components are $1/3, 1/3, 1/3$.

   Answer: \mathbfF(1, 1, 1) = \frac13\mathbfi + \frac13\mathbfj + \frac13\mathbfk

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