Triple integrals — setting up bounds in 3D

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Practice problems

1. Set up the triple integral for the volume of the solid E in the first octant bounded by the plane $z = x + y$, the plane $z = 0$, and the cylinder $x^2 + y^2 = 1$.
2. Set up the triple integral to find the mass of a cube with side length 2, centered at the origin, with density $\rho(x, y, z) = |x| + |y| + |z|$.
3. A solid is bounded by the plane $z = 0$, the cylinder $y^2 + z^2 = 1$, and the planes $x = 0$ and $x = 2$. Set up the triple integral for its volume using the order $dx\,dz\,dy$.

Solutions

1. Set up the triple integral for the volume of the solid E in the first octant bounded by the plane $z = x + y$, the plane $z = 0$, and the cylinder $x^2 + y^2 = 1$.

    - Identify the z bounds: the solid is bounded below by $z = 0$ and above by $z = x + y$.
    - Identify the shadow region R in the xy-plane: it is the quarter disk $x^2 + y^2 \leq 1$ where $x \geq 0$ and $y \geq 0$.
    - Convert the shadow region to polar coordinates: $0 \leq r \leq 1$ and $0 \leq \theta \leq \pi/2$.
    - Substitute $x = r\cos\theta$ and $y = r\sin\theta$ into the z bound: $z_{top} = r\cos\theta + r\sin\theta$.
    - Assemble the integral with the Jacobian r.

    Answer: \int_0ˆ\pi/2 \int_0ˆ1 \int_0ˆr(\cos\theta + \sin\theta) r \, dz \, dr \, d\theta

2. Set up the triple integral to find the mass of a cube with side length 2, centered at the origin, with density $\rho(x, y, z) = |x| + |y| + |z|$.

    - The cube is centered at the origin with side length 2, so the bounds for x, y, and z are all from -1 to 1.
    - Since the bounds are constants, the order of integration does not matter.
    - The density function is $\rho(x, y, z) = |x| + |y| + |z|$.
    - The integral is $\int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1}(|x| + |y| + |z|)\, dz\, dy\, dx$.

    Answer: \int_-1ˆ1 \int_-1ˆ1 \int_-1ˆ1 (—x— + —y— + —z—) \, dz \, dy \, dx

3. A solid is bounded by the plane $z = 0$, the cylinder $y^2 + z^2 = 1$, and the planes $x = 0$ and $x = 2$. Set up the triple integral for its volume using the order $dx\, dz\, dy$.

    - The order $dx\, dz\, dy$ means x is the innermost integral.
    - The x bounds are given as $x = 0$ to $x = 2$.
    - The remaining region is the shadow in the yz-plane, which is the semi-disk $y^2 + z^2 \leq 1$ where $z \geq 0$.
    - For a fixed y, z goes from 0 to $\sqrt{1 - y^2}$.
    - The outermost variable y ranges from -1 to 1.

    Answer: \int_-1ˆ1 \int_0ˆ\sqrt1-yˆ2 \int_0ˆ2 1 \, dx \, dz \, dy

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