

Setting up bounds II — swapping order

Lesson 23 · Module 2 — Multiple Integrals · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

Practice problems

1. Evaluate the integral $\int_0^1 \int_x^1 \sin(y^2) \, dy \, dx$ by swapping the order of integration.
2. Swap the order of integration for $\int_0^2 \int_{y/2}^1 e^{x^2} \, dx \, dy$ and evaluate.
3. Consider the integral $\int_0^1 \int_x^1 e^{y^2} \, dy \, dx$. Swap the order and evaluate.

Solutions

1. Evaluate the integral $\int_0^1 \int_x^1 \sin(y^2) dy dx$ by swapping the order of integration.

- Identify the region: $x \leq y \leq 1$ and $0 \leq x \leq 1$.
- Swap to type two: for a fixed y , x ranges from 0 to y .
- The outer bounds for y are 0 to 1.
- The new integral is $\int_0^1 \int_0^y \sin(y^2) dx dy$.
- Integrate with respect to x : $\int_0^1 [x \sin(y^2)]_0^y dy = \int_0^1 y \sin(y^2) dy$.
- Use u-substitution $u = y^2$, $du = 2y dy$: $\frac{1}{2} \int_0^1 \sin(u) du = [-\frac{1}{2} \cos(u)]_0^1 = \frac{1}{2}(1 - \cos(1))$.

Answer: $\frac{1}{2}(1 - \cos(1))$

2. Swap the order of integration for $\int_0^2 \int_{y/2}^1 e^{x^2} dx dy$ and evaluate.

- Region: $y/2 \leq x \leq 1$ and $0 \leq y \leq 2$.
- This is a triangle with vertices $(0,0)$, $(1,0)$, $(1,2)$.
- Swap to type one: x ranges from 0 to 1.
- For a fixed x , y ranges from 0 to $2x$.
- New integral: $\int_0^1 \int_0^{2x} e^{x^2} dy dx$.
- Inner integral: $[ye^{x^2}]_0^{2x} = 2xe^{x^2}$.
- Outer integral: $\int_0^1 2xe^{x^2} dx = [e^{x^2}]_0^1 = e - 1$.

Answer: $e - 1$

3. Consider the integral $\int_0^1 \int_x^1 e^{y^2} dy dx$. Swap the order and evaluate.

- Region: $x \leq y \leq 1$ and $0 \leq x \leq 1$.
- Swap to type two: y ranges from 0 to 1.
- For a fixed y , x ranges from 0 to y .
- New integral: $\int_0^1 \int_0^y e^{y^2} dx dy$.
- Inner integral: $[xe^{y^2}]_0^y = ye^{y^2}$.
- Outer integral: $\int_0^1 ye^{y^2} dy = [\frac{1}{2}e^{y^2}]_0^1 = \frac{1}{2}(e - 1)$.

Answer: $\frac{1}{2}(e - 1)$