Setting up bounds I — the region_slicer method

Lesson 22 · Module 2 — Multiple Integrals · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

Practice problems

1. Set up the iterated integral for the region R bounded by $y = x^2$ and $y = 4$ using vertical slices (Type I).
2. Set up the iterated integral for the region R bounded by $y = x$ and $y = x^2$ using horizontal slices (Type II).
3. Set up the iterated integral for the region R bounded by $y = e^x$, $y = 1$, and $x = 1$ using vertical slices.

Solutions

1. Set up the iterated integral for the region R bounded by $y = x^2$ and $y = 4$ using vertical slices (Type I).

   - Find intersection points: $x^2 = 4$ implies $x = -2$ and $x = 2$.
   - For a vertical slice, the bottom boundary is $y = x^2$ and the top boundary is $y = 4$.
   - The outer limits are the x-range from -2 to 2.

   Answer: \int_-2^2 \int_x^2^4 f(x, y) \, dy \, dx

2. Set up the iterated integral for the region R bounded by $y = x$ and $y = x^2$ using horizontal slices (Type II).

   - Find intersection points: $x = x^2$ implies $x = 0$ and $x = 1$. This means y also ranges from 0 to 1.
   - Solve boundaries for x: $y = x$ becomes $x = y$, and $y = x^2$ becomes $x = \sqrt{y}$ (for $x \geq 0$).
   - For a horizontal slice, the left boundary is $x = y$ and the right boundary is $x = \sqrt{y}$.

   Answer: \int_0^1 \int_y^\sqrty f(x, y) \, dx \, dy

3. Set up the iterated integral for the region R bounded by $y = e^x$, $y = 1$, and $x = 1$ using vertical slices.

   - The region is bounded by $y = e^x$ (top), $y = 1$ (bottom), and $x = 1$ (right).
   - The left intersection is where $e^x = 1$, which is $x = 0$.
   - Inner limits: y goes from 1 to e^x.
   - Outer limits: x goes from 0 to 1.

   Answer: \int_0^1 \int_1^e^x f(x, y) \, dy \, dx
