Implicit differentiation in several variables

Lesson 15 · Module 1 — Differentiation in Several Variables · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

---

Practice problems

1. Given the surface $x^3 + y^3 + z^3 + 6xyz = 1$, find the expression for $\frac{\partial z}{\partial x}$.
2. Find $\frac{\partial z}{\partial y}$ for the surface $\ln(z) + x^2 y + z = 10$ at the point $(1, 1, e)$.
3. For the surface $x^2 + 2y^2 + 3z^2 = 6$, show that the implicit derivative $\frac{\partial z}{\partial x}$ is equal to $-\frac{x}{3z}$.

Solutions

1. Given the surface $x^3 + y^3 + z^3 + 6xyz = 1$, find the expression for $\frac{\partial z}{\partial x}$.

   - Define $F(x, y, z) = x^3 + y^3 + z^3 + 6xyz - 1 = 0$.
   - Compute $F_x = 3x^2 + 6yz$.
   - Compute $F_z = 3z^2 + 6xy$.
   - Apply the formula $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$.

   Answer: $\frac{\partial z}{\partial x} = -\frac{3x^2+6yz}{3z^2+6xy} = -\frac{x^2+2yz}{z^2+2xy}$

2. Find $\frac{\partial z}{\partial y}$ for the surface $\ln(z) + x^2y + z = 10$ at the point $(1, 1, e)$.

   - Define $F(x, y, z) = \ln(z) + x^2y + z - 10 = 0$.
   - Compute $F_y = x^2$.
   - Compute $F_z = \frac{1}{z} + 1$.
   - At $(1, 1, e)$, $F_y = 1^2 = 1$ and $F_z = \frac{1}{e} + 1 = \frac{1+e}{e}$.
   - Apply $\frac{\partial z}{\partial y} = -\frac{1}{(1+e)/e} = -\frac{e}{1+e}$.

   Answer: $-\frac{e}{1+e}$

3. For the surface $x^2 + 2y^2 + 3z^2 = 6$, show that the implicit derivative $\frac{\partial z}{\partial x}$ is equal to $-\frac{x}{3z}$.

   - Define $F(x, y, z) = x^2 + 2y^2 + 3z^2 - 6 = 0$.
   - Compute $F_x = 2x$.
   - Compute $F_z = 6z$.
   - Apply the formula $\frac{\partial z}{\partial x} = -\frac{2x}{6z}$.
   - Simplify the fraction to get $-\frac{x}{3z}$.

   Answer: $\frac{\partial z}{\partial x} = -\frac{x}{3z}$
