The gradient I — steepest ascent

Lesson 12 · Module 1 — Differentiation in Several Variables · Multivariable & Vector Calculus · dailymaths.uk/vectorcalc

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Practice problems

1. Find the gradient vector ∇f for $f(x, y) = x^3 + y^3 - 6xy$ at the point $(1, 1)$.
2. For the function $f(x, y) = e^{2x-y}$, find the direction of steepest ascent at the point $(0, 0)$ and the value of the maximum slope.
3. A level curve of a function $f(x, y)$ is given by the circle $x^2 + y^2 = 25$. If the gradient at the point $(3, 4)$ is $\nabla f(3, 4) = \langle 6, 8 \rangle$, verify that the gradient is perpendicular to the level curve at that point.

Solutions

1. Find the gradient vector ∇f for $f(x, y) = x^3 + y^3 - 6xy$ at the point $(1, 1)$.

   - Compute the partial derivative with respect to x: $f_x = 3x^2 - 6y$.
   - Compute the partial derivative with respect to y: $f_y = 3y^2 - 6x$.
   - Evaluate f_x at $(1, 1)$: $3(1)^2 - 6(1) = -3$.
   - Evaluate f_y at $(1, 1)$: $3(1)^2 - 6(1) = -3$.
   - Combine into a vector: $\nabla f(1, 1) = \langle -3, -3 \rangle$.

   Answer: \langle -3, -3 \rangle

2. For the function $f(x, y) = e^{2x-y}$, find the direction of steepest ascent at the point $(0, 0)$ and the value of the maximum slope.

   - Compute $f_x = 2e^{2x-y}$ using the chain rule.
   - Compute $f_y = -e^{2x-y}$ using the chain rule.
   - Evaluate at $(0, 0)$: $f_x(0, 0) = 2e^0 = 2$ and $f_y(0, 0) = -e^0 = -1$.
   - The direction of steepest ascent is $\nabla f(0, 0) = \langle 2, -1 \rangle$.
   - The maximum slope is $|\nabla f(0, 0)| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$.

   Answer: \textDirection: \langle 2, -1 \rangle, \text Slope: \sqrt5

3. A level curve of a function $f(x, y)$ is given by the circle $x^2 + y^2 = 25$. If the gradient at the point $(3, 4)$ is $\nabla f(3, 4) = \langle 6, 8 \rangle$, verify that the gradient is perpendicular to the level curve at that point.

   - The level curve is $g(x, y) = x^2 + y^2 - 25 = 0$.
   - The tangent direction to the curve at $(3, 4)$ can be found by the vector $\langle -y, x \rangle = \langle -4, 3 \rangle$.
   - Check the dot product of the gradient $\langle 6, 8 \rangle$ and the tangent vector $\langle -4, 3 \rangle$.
   - Compute: $(6)(-4) + (8)(3) = -24 + 24 = 0$.
   - Since the dot product is zero, the gradient is perpendicular to the tangent of the level curve.

   Answer: \textVerified: \langle 6, 8 \rangle \cdot \langle -4, 3 \rangle = 0
